ON THE BIAS OF ADAPTIVE FIRST-ORDER RECURSIVE SMOOTHING

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ABSTRACT

In signal processing, first-order recursive smoothing is often used to determine the mean of a nonstationary random variable. In order to find a better compromise between the tracking speed and the variance of the estimate, adaptive smoothing factors have been proposed, e.g., for single-channel background noise power spectral density estimators. In this paper, the bias of recursive smoothing using adaptive smoothing functions is investigated. For adaptive functions that do not depend on the estimated mean an analytical derivation of the bias is given. For adaptive functions having a dependence on the recursively estimated mean, an iterative procedure is proposed which allows to approximately determine the bias with a sufficiently high precision.

***Index Terms*—** Estimation error, adaptive estimation, smoothing methods, IIR filters, speech enhancement

1. INTRODUCTION

In signal processing applications, a common problem is to estimate the mean of a random process, e.g., for computing the speech and noise power spectral densities (PSDs) required for Wiener filtering. In this example, these quantities can be interpreted as the mean of the speech and noise periodograms, respectively. For computing the mean of a random process, it is generally necessary to determine the ensemble average of the random process, which, however, can only be obtained for some theoretical cases. Thus, in many applications, the random process is often assumed to be ergodic so that the ensemble average can be replaced by a temporal smoothing. Further, as many signals can only be considered stationary over a limited time period, e.g., speech, the averaging is often performed over a short time-window. Therefore, first-order recursive smoothing is a commonly applied technique for estimating the mean of a random variable. It is specified as

$$\overline{x}_\ell = (1-\alpha)x_\ell + \alpha\overline{x}_{\ell-1}, \quad (1)$$

where x_ℓ is an observation of a random process and $\overline{x}_\ell$ the estimated mean. Further, $\alpha \in [0,1]$ denotes the smoothing constant and ℓ is the time index. The averaging in (1) is equivalent to computing an infinite sum over the past observations which are weighted by an exponential decaying window [1]. Here, the weight put on the recent observations is larger than for the past observations. Hence, the filter can track changes in the mean value where the tracking speed is controlled by the smoothing constant α. The tracking speed will be high if α is close to zero, whereas the variance of the estimate will decrease with increasing α. Thus, the choice of α is usually a trade-off between tracking speed and variance reduction.

As argued in [2], the variance of the estimate should be as small as possible while on the other hand the tracking should be as fast as possible. As these requirements cannot be met by a constant smoothing parameter α, the application of an adaptive smoothing factor is proposed in [2]. For this, an optimal time varying smoothing is derived by optimizing the quadratic error between the true and estimated mean. Adaptive smoothing functions have also been used in other single-channel noise PSD estimators, e.g., [3, Section 14.1.3] and implicitly also in [4]. Here, adaptive smoothing is employed to avoid speech leakage. In [3, Section 14.1.3], it is proposed to use a stronger smoothing for large a posteriori signal-to-noise ratios (SNRs). In [4], the value of the smoothing factor is implicitly adapted using the speech presence probability (SPP) and also grows with an increasing a posteriori SNR. In contrast to [3], a soft transition is implied here which arises from incorporating an estimate of the SPP into the noise PSD estimator.

Even though adaptive smoothing solves the issue of satisfying conflicting requirements, the resulting estimate will generally be biased as we will show in this paper. Given an adaptive function only depending on the unsmoothed observation x_ℓ of the random process and the distribution of x_ℓ, an analytical expression is derived for the expected value of $\overline{x}_\ell$ where the probability density function (PDF) of the smoothed quantity $\overline{x}_\ell$ is not required. Furthermore, based on this result, a method is proposed which can be used to approximately determine the bias if the adaptive function itself depends on the estimate $\overline{x}_\ell$. This method can therefore be used for many adaptive smoothing algorithms. In this paper we choose the adaptive smoothing functions in [3, Section 14.1.3] and [4], as well as a function similar to [2], as three practical examples taken from noise PSD estimation in single channel speech enhancement. In [2], another bias arises due to the usage of minimum-statistics which is addressed in [5].

The paper is organized as follows. First, we will derive an analytic expression for determining the expected value of a first-order filter incorporating an adaptive smoothing factor. Here, no dependence of the recursively smoothed estimate of the mean is assumed. Based on this result, a method is proposed which can be used to approximately determine the bias for adaptive functions that depend on the estimated mean itself. For the adaptive factors given in [3, 4] and a factor similar to [2], the required analytic expressions are derived and we will show that the proposed method gives a good estimate of the bias by comparing the results with the true deviations obtained from Monte-Carlo simulations.

2. EXPECTED VALUE OF ADAPTIVE RECURSIVE SMOOTHING

In this section, an analytic expression for the expected value of $\overline{x}_\ell$ is derived which can be used to determine the bias which occurs due to the usage of an adaptive smoothing factor $\alpha(x_\ell)$. First, we will assume that the adaptive smoothing factor $\alpha(x_\ell)$ only depends on the observation x_ℓ but not on the mean value $\overline{x}_\ell$. Based on these analytically solvable results, we will present a method which can be used to approximately determine the bias if the adaptive smoothing function depends on the estimated mean $\overline{x}_\ell$.

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2.1. Adaptive functions not depending on the estimated mean

For an adaptive function $\alpha(x_\ell)$ that can be assumed to be independent of the estimated mean $\overline{x}_\ell$ the first-order recursive filter, (1), can be written as

$$\overline{x}_\ell = [1-\alpha(x_\ell)]x_\ell + \alpha(x_\ell)\overline{x}_{\ell-1}. \tag{2}$$

Here, we assume that all x_ℓ are identically distributed and uncorrelated. Further, we assume that the stationary input x_ℓ will result in a stationary output $\overline{x}_\ell$. Experiments indicate that this property is sufficiently fulfilled for the adaptive smoothing factors considered in this paper. From this follows $\mathbb{E}\{\overline{x}_n\} = \mathbb{E}\{\overline{x}_m\}$, where $\mathbb{E}\{\cdot\}$ denotes the statistical expectation operator. Further, using the first assumption, the expected value $\mathbb{E}\{x_\ell \overline{x}_{\ell-1}\}$ can be written as $\mathbb{E}\{x_\ell\}\mathbb{E}\{\overline{x}_{\ell-1}\}$. Thus, by applying $\mathbb{E}\{\cdot\}$ to (2)

$$\mathbb{E}\{\overline{x}\} = \frac{\mathbb{E}\{x\} - \mathbb{E}\{x\alpha(x)\}}{1-\mathbb{E}\{\alpha(x)\}} \tag{3}$$

is obtained after rearranging the terms. As the random variables x_ℓ are assumed to be identically distributed, the time index ℓ is omitted. The result given in (3) depends only on the adaptive function $\alpha(x)$ and the PDF of x. If $\alpha(x)$ is set to α, i.e., a constant smoothing coefficient is used, it can be seen from (3) that the first-order filter gives an unbiased estimate of the random process.

2.2. Adaptive functions depending on the estimated mean

In this subsection, we will adapt the expression in (3) for obtaining an iterative algorithm which can be used to approximately determine the expected value $\mathbb{E}\{\overline{x}\}$ for the case where $\alpha(x,\overline{x})$ also depends on the recursively estimated mean $\overline{x}$. In this case, the quantity $\overline{x}$ influences the behavior of the adaptive smoothing factor which, in turn, again influences the estimation of $\overline{x}$. For noise PSD estimation, this kind of adaptation is most relevant, e.g. for the approaches [2, 3, 4].

The difficulty of including the quantity $\overline{x}$ within the computation is that $\overline{x}$ is a combination of random variables, namely all past x_ℓ. Therefore, $\overline{x}$ is a random variable itself which, additionally, is correlated with the previous estimates of $\overline{x}$. As it appears particularly difficult to derive an expression for the PDF of $\overline{x}$ and its correlations, we decide to simplify the problem by replacing $\overline{x}$ in the adaptive function by a fixed value ρ. Thus, we propose to determine the deviation from the true mean $\mathbb{E}\{x\}$ iteratively, where the estimated mean is fed back into the next iteration as

$$\rho_i = \frac{\mathbb{E}\{x\} - \mathbb{E}\{x\alpha(x,\rho_{i-1})\}}{1-\mathbb{E}\{\alpha(x,\rho_{i-1})\}}. \tag{4}$$

Here, ρ_i is the mean estimated for the ith iteration step whereas the initial condition is denoted by ρ_0. Even though this approach is motivated by the recursive update of $\overline{x}$, which in (2) is performed sample by sample, each step of the iteration, (4), considers all samples over an infinite time period. For obtaining an estimate of $\mathbb{E}\{\overline{x}\}$, the iteration is continued until it converges. We will demonstrate by the means of examples that the parameter ρ_0 does not influence the convergence of the iterative approach for the three adaptive smoothing algorithms [2, 3, 4]. The value of ρ_i after convergence can be interpreted as an equilibrium point where the system returns the same value as mean which is also used for the adaptive smoothing function.

3. SPECIFIC ADAPTIVE SMOOTHING FUNCTIONS

In this section, we will analyze the adaptive smoothing functions presented in [2, 3, 4] using the method described in Section 2. As all of these functions are used within noise PSD estimators, we assume that the noisy input signal is given by the superposition of a clean speech signal and a noise signal. The variable x can be understood as the periodogram of the noisy input signal while $\overline{x}$ represents the estimated noise PSD. Thus, we will follow the common assumption that the observed variable x follows an exponential distribution which is given by

$$f(x) = \begin{cases} (1/\mu)\exp(-x/\mu), & \text{if } x \geq 0, \\ 0, & \text{otherwise,} \end{cases} \tag{5}$$

where $\mu = \mathbb{E}\{x\}$. The choice of this PDF is motivated by the fact that the complex spectral coefficients of the discrete Fourier transform are often assumed to be normally distributed.

In the following subsections, we will first present the considered adaptive functions from [2, 3, 4] and derive the corresponding analytic expression of the expected value in (3). For this, we make use of the simplification described in Section 2.2 and replace $\overline{x}$ by the deterministic ρ. Further, we will show that the analytically derived bias is exact if the adaptive smoothing factor does not depend on the estimated mean $\overline{x}$ itself. For this, the analytically determined mean is compared to an estimate of the true bias obtained from Monte-Carlo simulations. For functions that depend on $\overline{x}$, we will analyze how large the difference between the true and the iteratively determined deviation, (4), is. Also for this comparison, Monte-Carlo simulations are used for obtaining an estimate of the true bias.

3.1. Two different smoothing factors

In [3, Section 14.1.3], a simple approach for estimating the background noise PSD has been proposed. Depending on a threshold value, a smoothing factor is chosen among two fixed values. A larger constant is used if the periodogram of the input signal is larger than the previously estimated background noise PSD while a smaller smoothing constant is used otherwise. The motivation of this approach is that observations having a large magnitude are likely to contain speech. Therefore, the tracking speed is reduced for increasing amplitudes in order to avoid speech energy leaking into the noise PSD estimate. The corresponding adaptive smoothing function is given by

$$\alpha_A(x,\rho) = \begin{cases} \alpha^\uparrow, & \text{if } x/\rho > 1, \\ \alpha^\downarrow, & \text{otherwise,} \end{cases} \tag{6}$$

where $\alpha^\downarrow$ and $\alpha^\uparrow$ are fixed constants between 0 and 1. The expected value $\mathbb{E}_A\{\overline{x}\}$, i.e., the solution to (3) given (6), can be derived as

$$\mathbb{E}_A\{\overline{x}\} = \mu \frac{(\alpha^\downarrow - 1)\exp(\lambda) + (\alpha^\uparrow - \alpha^\downarrow)(1+\lambda)}{(\alpha^\downarrow - 1)\exp(\lambda) + \alpha^\uparrow - \alpha^\downarrow}, \tag{7}$$

where λ is given by $\lambda = \rho/\mu$ and corresponds to the ratio between the assumed mean ρ in (6) and the true mean $\mu = \mathbb{E}\{x\}$ in (5).

3.2. Adaptive smoothing based on the SPP

The noise PSD estimator proposed in [4] uses an estimate of the SPP $P(H_1|x_\ell)$ in order to prevent a leakage of the speech power. Here, H_0 and H_1 denote the hypotheses that speech is absent and that speech is present, respectively. The SPP is determined for each time-frequency point using

$$P(H_1|x_\ell) = \left(1 + (1+\xi)\exp\left(-\frac{x_\ell}{\overline{x}_{\ell-1}}\frac{\xi}{\xi+1}\right)\right)^{-1}, \tag{8}$$

where ξ is an SNR that is expected if speech is present [4]. Further, the prior probabilities $P(H_0)$ and $P(H_1)$ are assumed to be equal. The SPP is used to obtain an estimate of the noise periodogram $\hat{d}_\ell$ as

$$\hat{d}_\ell = \{1 - P(H_1|x_\ell)\}x_\ell + P(H_1|x_\ell)\overline{x}_{\ell-1}, \tag{9}$$

which is smoothed using a fixed, non-adaptive, smoothing factor α

$$\overline{x}_\ell = (1-\alpha)\hat{d}_\ell + \alpha\overline{x}_{\ell-1} \tag{10}$$

to obtain an estimate of the background noise PSD.

Albeit no interpretation as adaptive smoothing has been given in [4], this function can be deduced by combining (9) and (10) which is then given as the factor multiplied by $\overline{x}_{\ell-1}$. Further, by replacing $P(H_1|x_\ell)$ by the expression in (8) and using the substitution ρ for $\overline{x}_{\ell-1}$

$$\alpha_B(x,\rho) = \alpha + \frac{1-\alpha}{1+(1+\xi)\exp(-x\xi/\{\rho[1+\xi]\})} \tag{11}$$

is obtained. The behavior of this adaptive function is similar to the adaptive smoothing factor proposed in [3, Section 14.1.3]. The function $\alpha_B(x,\rho)$ is close to one, i.e., the tracking speed is low, if the ratio x/ρ is large and is close to the fixed smoothing constant α otherwise.

The expected value $\mathbb{E}_B\{\overline{x}\}$ for the expression in (11) can be derived using the property of the geometric series [6, 1.112.1] and the analytic continuation property of the hypergeometric series [6, 9.130]. The result is

$$\mathbb{E}_B\{\overline{x}\} = \mu\frac{1-{}_3F_2[1,\zeta,\zeta;\zeta+1,\zeta+1;-(1+\xi)]}{1-{}_2F_1[1,\zeta;\zeta+1;-(1+\xi)]}, \tag{12}$$

where ${}_pF_q$ is the generalized hypergeometric function and ζ is

$$\zeta = \lambda\frac{\xi+1}{\xi}. \tag{13}$$

3.3. MMSE-optimal adaptive smoothing

Finally, we consider the following adaptive smoothing function

$$\alpha_C(x,\rho) = \frac{1}{1+(x/\rho-1)^2}. \tag{14}$$

This function is similar to the adaptive smoothing proposed in [2, Eq. (7)] to determine the PSD of the noisy input signal in the context of the minimum-statistics noise PSD estimator. It has been derived as the minimum mean square error between the true and the estimated mean given the estimated mean of the previous time step. The motivation for this is to solve the issue of conflicting requirements. On one hand, the tracking of the background noise PSD needs to be fast so that the smoothing factor should be close to zero. On the other hand, the variance of the estimate should be as small as possible so that the smoothing should be as close to one as possible [2].

For this function, $\mathbb{E}_C\{\overline{x}\}$ can be derived by using the method of partial fraction expansion and [6, 2.325.1] which results in

$$\mathbb{E}_C\{\overline{x}\} = \mu\frac{1-\sqrt{2}e^{-\lambda}\lambda^2(\pi\sin(\tilde{\lambda})-\Re\{e^{-\jmath\tilde{\lambda}}\mathrm{Ei}[(1+\jmath)\lambda]\})}{1-\lambda e^{-\lambda}(\pi\cos(\lambda)-\Im\{e^{-\jmath\lambda}\mathrm{Ei}[(1+\jmath)\lambda]\})}, \tag{15}$$

where $\tilde{\lambda}$ is given by $\tilde{\lambda} = \lambda + \pi/4$ and $\mathrm{Ei}(x) = \int_{-x}^{\infty} e^{-x}/x\,dx$ is the exponential integral. Further, $\jmath^2 = -1$ is the imaginary unit while $\Re\{\cdot\}$ and $\Im\{\cdot\}$, respectively, denote the real and imaginary part of a complex number.

From (7), (12) and (15), it can be seen that the ratio $\mathbb{E}\{\overline{x}\}/\mu$ depends only on the ratio $\lambda = \rho/\mu$. Consequently, the ratio between the true mean $\mathbb{E}\{x\} = \mu$ and the expected value $\mathbb{E}\{\overline{x}\}$ is fixed and does not depend on the scaling of x. Therefore, for compensating the bias, a simple correction factor can be used which is given by

$$R = \frac{\mu}{\mathbb{E}\{\overline{x}\}}. \tag{16}$$

	$\alpha_A(x,\rho)$; (6)	$\alpha_B(x,\rho)$; (11)	$\alpha_C(x,\rho)$; (14)
R	2.37	1.16	0.72
R_{MC}	2.36	1.16	0.72

Table 1: Correction factor $R = \mu/\mathbb{E}\{\overline{x}\}$ under the assumption that $\rho = \mu$ for the adaptive smoothing functions in (6), (11) and (14). The quantity R_{MC} corresponds to the correction factor obtained using Monte-Carlo simulations using 10^6 realizations.

If the smoothing function does not depend on $\overline{x}$, the bias compensation is achieved by computing R with (16) and by applying Algorithm 1. Here, $\check{\overline{x}}$ denotes the corrected version of $\overline{x}$. For adaptive smoothing factors depending on $\overline{x}$, R is determined off-line using Algorithm 2. Here, also μ can be set to an arbitrary value because this factor cancels out in line six of the algorithm.

4. EVALUATION

In this section the proposed method is evaluated by comparing the bias obtained by solving (3) or by using Algorithm 2 to values obtained from Monte-Carlo simulations. Here, the two cases are considered that the adaptive smoothing functions do and do not depend on $\overline{x}$.

4.1. Smoothing functions not depending on $\overline{x}$

In this subsection, we will show that the results obtained from (3) are exact if the adaptive smoothing function does not depend on the estimated mean $\overline{x}$. Consequently, it is assumed that $\overline{x}$ is equal to the fixed value ρ which is set to the true mean μ for this analysis. In this evaluation, the default values for the parameters of the adaptive functions (6), (11), and (14) are employed. In accordance to [3, Section 14.1.3], the parameters $\alpha^{\uparrow} = 0.9995$ and $\alpha^{\downarrow} = 0.9$ are used for (6). For the SPP based smoothing function in (11), ξ is set to 15 dB, which was obtained by minimizing the Bayesian risk [7]. The fixed smoothing constant α is set to 0.8 as in [4]. No parameters are required for (14). The correction factor R, (16), is computed using (7), (12) and (15).

These results are verified by Monte-Carlo simulations where the mean $\mathbb{E}\{\overline{x}\}$ is estimated from a large sample of an exponentially distributed random variable. This can be understood as an input signal that consists only of a stationary noise component. For this, the sample is used as input of the first-order recursive filter, (2), where the adaptive smoothing factor is replaced by the respective adaptive function, i.e. (6), (11) or (14). For the estimation of $\mathbb{E}\{\overline{x}\}$ the mean of the filtered

Algorithm 1 Proposed algorithm for bias compensation.

1: **for all** observations x_ℓ **do**
2: Perform smoothing:
$\overline{x}_\ell = [1-\alpha(x_\ell,\overline{x}_{\ell-1})]x_\ell + \alpha(x_\ell,\overline{x}_{\ell-1})\overline{x}_{\ell-1}$.
3: Compensate bias: $\check{\overline{x}}_\ell = R\overline{x}_\ell$.
4: **end for**

Algorithm 2 Iterative bias estimation for adaptive functions depending on $\overline{x}$.

1: $i \leftarrow 0$, $\rho_0 \leftarrow 1$, $\mu \leftarrow 1$.
2: **while** convergence criterion for ρ_i is not met **do**
3: Obtain ρ_{i+1} using (4). The solutions for the adaptive functions in [2, 3, 4] are given in (7), (12), and (15), respectively.
4: $i \leftarrow i+1$
5: **end while**
6: Compute compensation factor: $R = \mu/\rho_i$, (16).

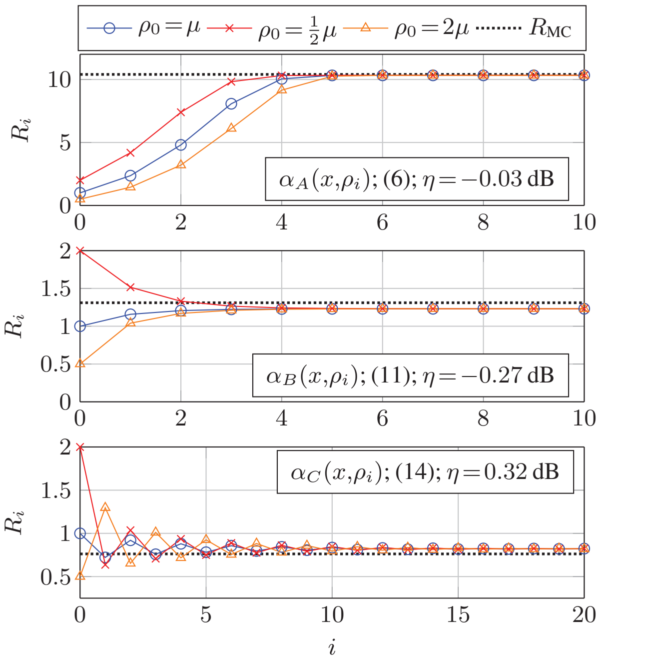

Figure 1: Bias correction factor $R_i = \mu/\rho_i$ computed for each iteration step in Algorithm 1 given the adaptive functions in (7), (12), (15) and the true bias correction term R_{MC} obtained from Monte-Carlo simulations with 10^6 realizations.

output samples is computed by temporal averaging. The result is used to determine a simulated correction factor R_{MC} analogous to (16) which is used for comparison. For this analysis, a sample size of 10^6 realizations has been used. The outcome is shown in Table 1.

The analytically calculated values R assort well with simulated quantities R_{MC} which indicates that the bias can be determined exactly using the respective solutions for (3). Further, the results in Table 1 show that due to their adaptive character, all adaptive smoothing functions are biased even though the fixed parameter ρ has been set to the true mean μ. As the functions $\alpha_A(x,\rho)$, (6), and $\alpha_B(x,\rho)$, (11), increase the smoothing for large values of x, the influence of these values becomes smaller on the estimated mean which results in an underestimation. Thus, the correction factors for these function is larger than one. The opposite effect leads to the overestimation which can be observed for the adaptive function $\alpha_C(x,\rho)$, (14). Here, a stronger smoothing is applied to small values x which, correspondingly, have a smaller influence on the estimated mean in comparison to large values. As a consequence, the mean is overestimated and a correction factor $R < 1$ needs to be applied.

4.2. Smoothing functions depending on $\overline{x}$

Here, the dependence of the adaptive smoothing function $\alpha(x,\overline{x})$ on the estimated mean $\overline{x}$ in (6), (11), (14) will be considered. For this, the deviation is estimated using the iterative method given in Algorithm 2 and compared to the true bias which is obtained using Monte-Carlo simulations. For the simulations, the procedure described in Section 4.1 is used to determine an estimate of $\mathbb{E}\{\overline{x}\}$. Here, however, $\overline{x}$ is employed in the feedback loop instead of the fixed value ρ. The result of each iteration step is shown as bias correction $R_i = \mu/\rho_i$ in Figure 1. Additionally, the correction factor R_{MC} obtained from the Monte-Carlo simulations is depicted. The deviation $\eta = R/R_{\mathrm{MC}}$ is used as quantitative evaluation criterion. Here, R denotes the bias correction term obtained using the converged value of ρ_i as in Algorithm 2.

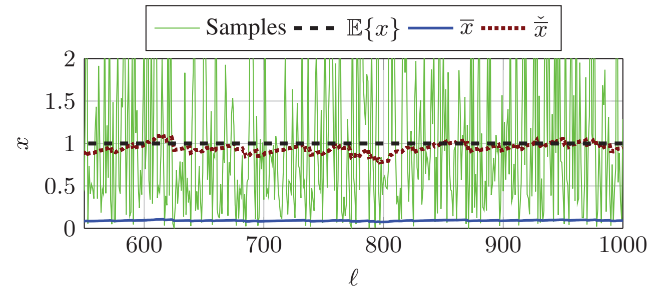

Figure 2: Time-course of the sample x, the estimated mean $\overline{x}$ with and without bias correction obtained using the adaptive function $\alpha_A(x,\overline{x})$, (6), [3, Section 14.1.3] and the true mean.

The results show that the iteration converges for all considered smoothing functions after 10 to 15 steps and that the value obtained after convergence is independent of the initial condition ρ_0. Further, the iteratively determined bias corresponds well with the Monte-Carlo simulations. For $\alpha_A(x,\rho_i)$ the iteratively determined correction factor R is nearly identical to the true one. Here, the ratio η is -0.03 dB. For $\alpha_B(x,\rho_i)$, (11), and $\alpha_C(x,\rho_i)$, (14), a small difference between these two values can be observed where the iteratively determined bias is usually less extreme than the true one. Here, η is -0.27 dB for $\alpha_B(x,\rho_i)$, (11), and 0.32 dB for $\alpha_C(x,\rho_i)$, (14). This difference can be explained by the fact that $\overline{x}$ is not considered as a random variable and also its correlations are omitted. Generally, for the adaptive functions regarded in this analysis, the accuracy of the iterative procedure, (4), gives a good estimate of the deviation which is caused by the adaptive smoothing.

Figure 2 shows an example where the adaptive smoothing function proposed in [3, Section 14.1.3] is used to estimate the mean of an exponentially distributed sample where the default parameters are used. This example is chosen as the effect of the bias compensation is most drastic here. Besides the true mean μ and the realizations x, the average $\overline{x}$ and the corrected average $\breve{\overline{x}}$ obtained by applying Algorithm 1 are shown. Without any correction, the time-course of the filtered input samples shows a large deviation from the true mean. Here, the mean is underestimated by 10.2 dB. The underestimation is less extreme, when lower values for $\alpha^{\uparrow}$ are chosen. Using the factor R, the bias can be corrected which results in an estimate much closer to the true mean. Correspondingly, also this example shows that the proposed iterative procedure is able to assess the bias with a sufficiently high precision and thus makes it possible to correct the bias inferred by the usage of the adaptive smoothing.

5. CONCLUSIONS

In this paper, the bias of first-order recursive filtering with adaptive smoothing factors has been investigated. The focus has been laid on adaptive functions used in single-channel background noise PSD estimators, namely [3, 4] and a function similar to [2]. Under the assumption that the adaptive functions do not depend on the estimated average, the bias can be analytically determined. It could be shown that the bias does not depend on the scale of the smoothed input quantity if an exponentially distributed random variable is assumed. Further, an iterative method based on the analytical solution has been proposed which can be used to determine the bias if the adaptive function does depend on the estimated mean. Even though this method can only be considered as an approximation, the evaluations show that the bias can be determined with a sufficiently high precision.

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