SPARSE RECONSTRUCTION OF QUANTIZED SPEECH SIGNALS

Christoph Brauer[⋆] *Timo Gerkmann*[†] *Dirk Lorenz*[⋆]

[⋆] Technische Universität Braunschweig, Germany
[†] Technicolor Research & Innovation, Hannover, Germany

ABSTRACT

We propose sparse reconstruction techniques to improve the quality and / or reduce the bit-rate of standard speech coders. To that end, we assume signal sparsity in some transform domain and formulate the problem of reconstructing the original signal in terms of constrained ℓ_1-norm minimization. We use modern primal-dual methods in order to solve the resulting non-smooth convex optimization problem. Experiments show that with the proposed sparse reconstruction method the instrumentally predicted speech quality can be largely improved.

Index Terms— Speech coding, quantization, compressed sensing, optimization methods.

1. INTRODUCTION

Speech coding describes how analog speech signals can efficiently be represented in the digital domain, for instance for storage or transmission. The goal of research in speech coding is to find algorithms that give the best possible trade-off between computational complexity for encoding and decoding, the required data-rate, algorithmic latency, and speech quality [1]. The field has traditionally been driven by applications in telecommunications. As such, the requirements and capacity on the sender and receiver side—both being telephones—were usually rather symmetric.

In recent years wireless acoustic sensor networks attract increasing interests in the audio and acoustics communities. The idea of wireless acoustic sensor networks is that many cheap, small, and potentially battery driven acoustic sensors are spread through a room or even a house of interest. In contrast to traditional fixed microphone arrays the essential benefit of a sensor network is that chances are higher that a sensor/microphone is close to the source of interest such that the source can be captured at a higher signal-to-noise ratio. To process the many signals captured by the acoustic sensor network, two possibilities exist. Either the processing can be done in a distributed fashion [2], or the signals can be sent to a central processor, called *fusion center*, where all processing is done [3]. In this paper, we focus on the latter. In contrast to traditional telecommunications, in such a scenario, the computational resources between sender and receiver are quite unbalanced. As the sender is small and possibly battery driven its computational capacity is limited. On the other hand, the receiving fusion center can be very powerful.

The goal of this paper is therefore to find a speech codec that results in a low power demand on the sender side while the computational complexity for decoding at the receiver can be significantly higher. To ensure a low power demand at the sender we need both a low computational complexity and also a bit-rate that is as low as possible. The lowest complexity for encoding at a low data-rate is possibly to simply quantize the signal with a low word length, i.e. well below 8 Bit per sample. To improve quality, A-law compression can be applied prior to quantization [1]. If such a low bit-rate signal is reconstructed at the receiver by simple interpolation, the resulting quality would of course be poor. Thus in this paper, we propose to reconstruct the signal quantized at a low bit-rate at the receiver by exploiting the sparsity of the signal in some transform domain and the prior knowledge that the original signal sample must lie within the respective quantization interval. Our approach is conceptually related to the idea of compressed sensing [4, 5]. There a signal that is sparse in some transform domain shall be captured by a number of linear measurements that is much smaller than the ambient dimension of the signal. In our situation, acoustic signals shall be captured and transmitted in real time and latency of the method has to be taken into account. Hence, we will stick to a simple Nyquist sampling, and the compression is not done by a *small number of measurements*, but by coarse quantization, i.e. a *low complexity of each measurement*. The similarity between compressed sensing and our approach is, that the inherent low complexity of the signal (i.e. sparsity in some transform domain) shall be leveraged for a better signal reconstruction. De-quantization in combination with compressed sensing has also been treated in [6, 7] however with a different approach and for different applications.

The paper is structured as follows. After defining the quantization functions in Section 2, in Section 3 we describe the reconstruction approach, and Section 4 reports numerical results. Finally, Section 5 draws conclusions.

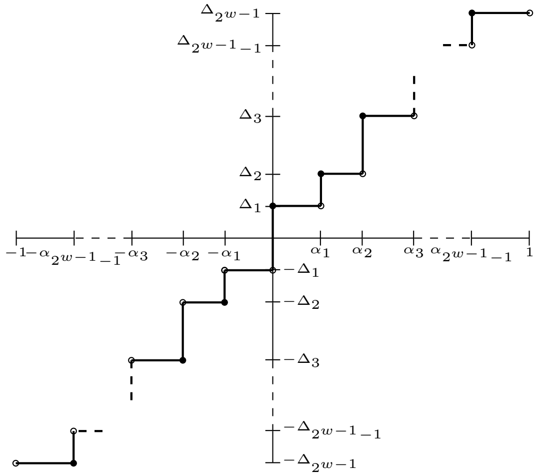

Fig. 1: Example for a quantization function of the form (1).

2. QUANTIZATION FUNCTIONS

Let $w \in \mathbf{N}$ denote the word length, i.e. the number of bits available for quantization, and $\alpha_0, \ldots, \alpha_{2^{w-1}} \in \mathbf{R}$ be the quantization levels such that

$$0 =: \alpha_0 < \alpha_1 < \cdots < \alpha_{2^{w-1}-1} < \alpha_{2^{w-1}} := 1.$$

Then the intervals $I_l := [\alpha_{l-1}, \alpha_l)$ ($l = 1, \ldots, 2^{w-1}$) form a partition of the interval $[0, 1)$. Moreover, let $0 < \Delta_l$ for $l = 1, \ldots, 2^{w-1}$ with $\Delta_l \neq \Delta_{l'}$ for $l \neq l'$. Finally, we define $I := (-1, 1)^N$ and let $\operatorname{sign}^+(x)$ be 1 for $x \geq 0$ and -1 for $x < 0$. For a sampled speech signal $\mathbf{f} \in I^N$, we consider quantization functions $Q : I^N \to I^N$ of the form

$$Q(\mathbf{f})_j := \operatorname{sign}^+(\mathbf{f}_j)\Delta_l \quad \text{if} \quad |\mathbf{f}_j| \in I_l \tag{1}$$

where $\mathbf{f}_j$ denotes the j-th component of $\mathbf{f}$ and $Q(\mathbf{f})_j$ denotes the j-th component of the quantized signal. From the above definition, it follows that Q is odd in each component and that it maps the interval I to 2^w different quantization levels, cf. Figure 1.

3. RECONSTRUCTION APPROACH

Suppose that a quantized speech signal $Q(\mathbf{f})$ is given and we seek to find a signal $\mathbf{x}$ that approximates the underlying true signal $\mathbf{f}$ as well as possible. We aim to find the true signal among all signals $\mathbf{x}$ that would give the same quantized signal as we observed, i.e. we restrict the search space to all signals $\mathbf{x}$ such that $Q(\mathbf{x}) = Q(\mathbf{f})$. However, this search is still hopeless without additional information since there are still infinitely many possibilities to choose $\mathbf{x}$.

The crucial assumption for our reconstruction approach is that the sought after signal $\mathbf{x}$ is sparse in some known transform domain, i.e. we have a transform matrix $\Psi \in \mathbf{R}^{N \times N}$ and expect that there exists a sparse vector $\mathbf{a} \in \mathbf{R}^N$ that satisfies $\mathbf{x} = \Psi\mathbf{a}$.

Consequently, we search a sparse coefficient vector $\mathbf{a}$ that satisfies $Q(\Psi\mathbf{a}) = Q(\mathbf{f})$. As known from compressed sensing [4, 5] this is achieved by ℓ_1-norm minimization, namely by solving

$$\min_{\mathbf{a} \in \mathbf{R}^N} \quad \|\mathbf{a}\|_1 \quad \text{s.t.} \quad Q(\Psi\mathbf{a}) = Q(\mathbf{f}). \tag{2}$$

Now it turns out that the non-linear constraint $Q(\Psi\mathbf{a}) = Q(\mathbf{f})$ has a convex and even linear reformulation: From the definition of Q it follows that $Q(\Psi\mathbf{a})_j = Q(\mathbf{f})_j$ if and only if $(\Psi\mathbf{a})_j$ is in the respective quantization interval, i.e. $Q(\mathbf{f})_j = \pm\Delta_l$ if and only if

$$(\Psi\mathbf{a})_j \in \pm I_l. \tag{3}$$

3.1. Uniform Quantization

We consider the case where Q is a mid-rise uniform quantization function, i.e. the quantization intervals satisfy $\Delta_l - \Delta_{l-1} = \Delta$ in (1). Depending on the word length w, we set $\Delta := 2^{-w+1}$ and have

$$Q(\mathbf{f})_j = Q_\Delta(\mathbf{f})_j := \operatorname{sign}^+(\mathbf{f}_j)\,\Delta\left(\left\lfloor \tfrac{|\mathbf{f}_j|}{\Delta} \right\rfloor + \tfrac{1}{2}\right).$$

Notice that with $\Delta_l := (l - \frac{1}{2})\Delta$ and $I_l := [(l-1)\Delta, l\Delta)$ this is exactly of the form (1). Hence, (3) simply becomes $|(\Psi\mathbf{a})_j - Q_\Delta(\mathbf{f})_j| \leq \frac{\Delta}{2}$. We conclude that in this particular case (2) turns out to be

$$\min_{\mathbf{a} \in \mathbf{R}^N} \quad \|\mathbf{a}\|_1 \quad \text{s.t.} \quad \|\Psi\mathbf{a} - Q_\Delta(\mathbf{f})\|_\infty \leq \tfrac{\Delta}{2}. \tag{4}$$

3.2. Non-Uniform Quantization

Let $C : I \to I$ be odd, continuous and strictly monotonically increasing. We investigate non-uniform quantization functions of the form $Q(x) = Q_\Delta(C(x))$, wherein Q_Δ is a mid-rise uniform quantization function as introduced in Section 3.1. Moreover, for ease of notation, we use $C(\mathbf{f})_j = C(\mathbf{f}_j)$ in case $\mathbf{f} \in \mathbf{R}^N$.

At first, it is not hard to see that $Q(\mathbf{f})_j = \pm\Delta_l$ if $|\mathbf{f}_j| \in C^{-1}(I_l)$ (and equal to $+\Delta_l$ for $\mathbf{f}_j \geq 0$ and $-\Delta_l$ for $\mathbf{f}_j < 0$), i.e.

$$Q(\mathbf{f})_j = \operatorname{sign}^+(\mathbf{f}_j)\Delta_l \quad \text{if} \quad |\mathbf{f}_j| \in C^{-1}(I_l)$$

which shows that Q can be written according to (1). Analogous to above, constraint (3) turns into

$$(\Psi\mathbf{a})_j \in [C^{-1}(Q_\Delta(\mathbf{f})_j - \tfrac{\Delta}{2}), C^{-1}(Q_\Delta(\mathbf{f})_j + \tfrac{\Delta}{2})].$$

Thus, writing $\alpha := C^{-1}(Q_\Delta(C(\mathbf{f})) - \frac{\Delta}{2})$ as well as $\beta := C^{-1}(Q_\Delta(C(\mathbf{f})) + \frac{\Delta}{2})$, the adaption of (2) to non-uniform quantization functions is

$$\min_{\mathbf{a} \in \mathbf{R}^N} \quad \|\mathbf{a}\|_1 \quad \text{s.t.} \quad \alpha \leq \Psi\mathbf{a} \leq \beta. \tag{5}$$

$$\mathbf{a}^{i+1} = \text{prox}_{\tau\|\cdot\|_1}(\mathbf{a}^i - \tau\Psi^T\mathbf{y}^i)$$
$$\overline{\mathbf{a}}^{i+1} = 2\mathbf{a}^{i+1} - \mathbf{a}^i$$
$$\mathbf{y}^{i+1} = \text{prox}_{\sigma I_\Omega^*}(\mathbf{y}^i + \sigma\Psi\overline{\mathbf{a}}^{i+1})$$

Algorithm 1: Primal-dual iteration for (7) with prox defined in (8).

In our subsequent numerical experiments, we use the well-known A-law compression function $C = C_A$. For some fixed $A \geq 1$, this is given by (e.g. [1])

$$C_A(x) := \begin{cases} \text{sign}(x)\,\frac{1+\ln(A|x|)}{1+\ln(A)} & \text{if} \quad |x| \geq \frac{1}{A} \\ \text{sign}(x)\,\frac{A|x|}{1+\ln(A)} & \text{else} \end{cases}.$$

The related inverse is given by

$$C_A^{-1}(y) := \begin{cases} \text{sign}(y)\,\frac{e^{|y|(1+\ln(A))-1}}{A} & \text{if} \quad |y| \geq \frac{1}{1+\ln(A)} \\ \frac{(1+\ln(A))y}{A} & \text{else} \end{cases}.$$

3.3. Algorithmic Framework

Both (4) and (5) are non-smooth constrained convex optimization problems. We reformulate the problems with *indicator functions*: For any convex set $\Omega \subseteq \mathbf{R}^N$, the related indicator function $I_\Omega : \mathbf{R}^N \to \mathbf{R}_\infty$ is defined by

$$I_\Omega(\mathbf{x}) := \begin{cases} 0 & \text{if} \quad x \in \Omega \\ \infty & \text{else} \end{cases}.$$

Further defining $\Omega_2 := \{\mathbf{x} \in \mathbf{R}^N : \|\mathbf{x} - Q_\Delta(\mathbf{f})\|_\infty \leq \frac{\Delta}{2}\}$ and $\Omega_3 := \{\mathbf{x} \in \mathbf{R}^N : \alpha \leq \mathbf{x} \leq \beta\}$, we can rewrite (4) and (5) as

$$\min_{\mathbf{a}\in\mathbf{R}^N} \quad \|\mathbf{a}\|_1 \quad + \quad I_\Omega(\Psi\mathbf{a}) \tag{6}$$

with $\Omega = \Omega_2$ and $\Omega = \Omega_3$, respectively. Using the convex conjugate I_Ω^* of I_Ω [8], (6) can be rewritten as

$$\min_{\mathbf{a}\in\mathbf{R}^N}\max_{\mathbf{y}\in\mathbf{R}^N} \quad \|\mathbf{a}\|_1 \quad + \quad \langle\Psi\mathbf{a},\mathbf{y}\rangle \quad - \quad I_\Omega^*(\mathbf{y}). \tag{7}$$

The latter is a convex-concave saddle-point problem which can be tackled by different optimization methods. In our experiments, we use the primal-dual algorithm proposed in [9]. The algorithm relies on the so-called *proximal operators* [8]: For a convex function F defined on $\mathbf{R}^N$, $\lambda > 0$ and $\mathbf{x} \in \mathbf{R}^N$ it holds that

$$\text{prox}_{\lambda F}(\mathbf{x}) := \underset{\mathbf{y}\in\mathbf{R}^N}{\text{argmin}}\, \lambda F(\mathbf{y}) + \|\mathbf{x} - \mathbf{y}\|_2^2/2. \tag{8}$$

Given a primal iterate $\mathbf{a}^i$ and a dual iterate $\mathbf{y}^i$, the $(i+1)$-th iteration of this algorithm applied to (7) consists of the three steps displayed in Algorithm 1 and only needs the proximal operators for $F(\mathbf{a}) = \tau\|\mathbf{a}\|_1$ and $G(\mathbf{y}) = \sigma I_\Omega^*(\mathbf{y})$. The first one is the so-called *soft thresholding operator* given by

$$\text{prox}_{\tau\|\cdot\|_1}(\mathbf{a})_j = \text{sign}(\mathbf{a}_j)\max(|\mathbf{a}_j| - \tau, 0).$$

And the proximal operators $\text{prox}_{\sigma I_\Omega^*}$ for $\Omega = \Omega_2, \Omega_3$, respectively are

$$\text{prox}_{\sigma I_{\Omega_2}^*}(\mathbf{y}) = \text{prox}_{\frac{\sigma\Delta}{2}\|\cdot\|_1}(\mathbf{y} - \sigma Q_\Delta(\mathbf{f}))$$

and

$$\text{prox}_{\sigma I_{\Omega_3}^*}(\mathbf{y})_j = \begin{cases} \mathbf{y}_j - \sigma\alpha_j & \text{if} \quad \mathbf{y}_j < \sigma\alpha_j \\ 0 & \text{if} \quad \mathbf{y}_j \in \sigma[\alpha_j, \beta_j] \\ \mathbf{y}_j - \sigma\beta_j & \text{if} \quad \mathbf{y}_j > \sigma\beta_j \end{cases},$$

respectively. Algorithm 1 converges as soon as the step-sizes $\tau, \sigma > 0$ fulfill $\tau\sigma\|\Psi\|^2 < 1$, see [9].

4. NUMERICAL EXPERIMENTS

In our experiments, we investigate the impact of Algorithm 1 on 720 sentences from the IEEE corpus provided in [10] consisting of male speech and sampled at 16 kHz.

As a first step, we quantize a speech signal $\mathbf{f}$ using a uniform or non-uniform quantization function as described in sections 3.1 and 3.2, respectively. Then the signal is split into overlapping sub-signals $\mathbf{f}^t$ whereupon we employ Algorithm 1 adapted to the respective quantization function. Finally, overlapping parts of the sub-solutions $\mathbf{x}^t$ are averaged in order to obtain the reconstructed signal $\mathbf{x}$.

Thereby, we assume that each sub-solution $\mathbf{x}^t$ has a sparse representation in terms of the discrete cosine basis, i.e. its discrete cosine transform $\text{DCT}(\mathbf{x}^t) = \mathbf{a}^t$ is sparse. Consequently, we have $\mathbf{x}^t = \text{IDCT}(\mathbf{a}^t)$ and use $\Psi = \text{IDCT}$ in Algorithm 1.

In order to split $\mathbf{f}$, we fix the size $n \leq N$ of the sub-signals as well as a shift length $s \leq n$. Therewith, the t-th sub-signal is given by

$$\mathbf{f}^t = (\mathbf{f}_{(t-1)s+1}, \ldots, \mathbf{f}_{(t-1)s+n}).$$

Considering that necessarily $(t-1)s + n \leq N$, we obtain sub-signals for $t = 1, \ldots, \lfloor\frac{N-n}{s}\rfloor + 1$.

We employ the Perceptual Evaluation of Speech Quality instrumental measure as provided by Loizou in [10] (PESQL) to validate the speech quality of the reconstucted signals. In case of a uniform quantization function, we compare the PESQL value of $\mathbf{x}$ to the PESQL value of the quantized signal $Q_\Delta(\mathbf{f})$. In case of an A-law quantization function, we compare with the PESQL value of the standard reconstruction $C_A^{-1}(Q_\Delta(C_A(\mathbf{f})))$.

Figure 2 displays average results over 720 male speech signals from the IEEE speech database. We performed the experiments outlined above using $n = 1024$, $s = 256$, word lengths $w = 2, \ldots, 8$ and a fixed number of 25 iterations in Algorithm 1. In general, 25 iterations are not sufficient to solve (4) and (5), respectively. Nevertheless, we observed that 25 iterations are enough to obtain a remarkably higher

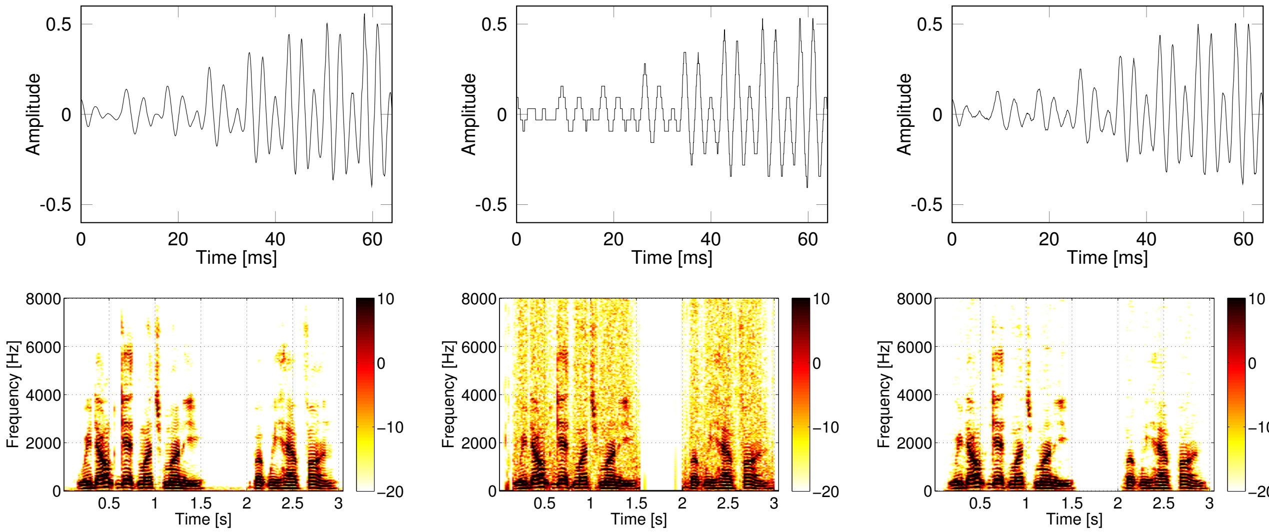

Fig. 3: Time-domain snippets of 64 ms length (top) and spectrograms (bottom) of clean (left), quantized (middle) and reconstructed (right) speech. The snippets are taken at time 0.2s. The sampling rate is 16 kHz and the word length for the quantized speech (middle) $w = 5$ Bit.

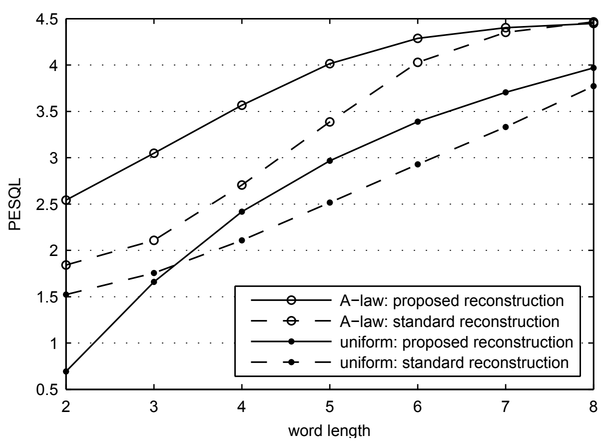

Fig. 2: Average PESQL values obtained in experiments with 720 speech signals from the IEEE corpus using $n = 1024$, $s = 256$, word lengths $w = 2, \ldots, 8$ and 25 iterations in Algorithm 1, compared to average PESQL values of the associated standard reconstructions $C_A^{-1}(Q_\Delta(C_A(\mathbf{f})))$ and $Q_\Delta(\mathbf{f})$, respectively.

PESQL value. Figure 3 shows time snippets as well as spectrograms of the results for the seventh signal from the 48th list in the database ("We don't get much money, but we have fun.") under the same algorithmic setting as described above.

Our implementation in MATLAB on an Intel®Core™I7 with 1.90GHz and 3.7GB RAM with 25 iterations could reconstruct a signal in less computational time than the signal length and hence, is amenable for real time performance. It should also be noted that already ten iterations lead to a significant improvement of the PESQL values while using 50 instead of 25 iterations only leads to a further minor improvement.

Notice that these results are obtained for $n = 1024$ and $s = 256$. A higher value of n increases the size of the sub-signals and thus the computational time for a fixed number of iterations. At the same time, a higher value of n can decrease the number of sub-problems to be solved. As well, a higher value of s tendencially decreases the number of sub-problems but also the number of sub-signals that overlap each sample.

5. CONCLUSIONS

In wireless acoustic sensor networks the power available for data encoding and transmission is often much lower than at the fusion center, where the signals are decoded and processed. Therefore, in this paper we proposed to encode a speech signal by quantizing the time domain samples with a low number of bits. While this coding scheme is computationally cheap, at low bit-rates traditional decoding schemes would yield a rather poor quality. Instead, we propose to exploit both the fact that speech signals are sparse in some transform domain and that we know that the unknown true speech sample lies within a given quantization interval. To solve the respective non-smooth optimization problem we use Chambolle-Pock's primal-dual method. This method leads to significant improvement of the PESQL and moreover, can be implemented in real time. Our results show that even the basic setup with the DCT, a rectangular window and only 25 iterations leads to this significant improvement of the PESQL value. Hence, further improvement is to be expected by fine-tuning of these ingredients.

6. REFERENCES

[1] P. Vary and R. Martin, *Digital Speech Transmission: Enhancement, Coding And Error Concealment*. Chichester, West Sussex, UK: John Wiley & Sons, 2006.

[2] Y. Zeng and R. C. Hendriks, "Distributed delay and sum beamformer for speech enhancement via randomized gossip," *IEEE Transactions on Audio, Speech, and Language Processing*, vol. 22, no. 1, pp. 260–273, Jan. 2014.

[3] A. Bertrand and M. Moonen, "Distributed adaptive node-specific signal estimation in fully connected sensor networks part i : Sequential node updating," *IEEE Transactions on Signal Processing*, vol. 58, no. 10, pp. 5277–5291, Oct. 2010.

[4] E. J. Candès, "Compressive sampling," in *Proc. International Congress of Mathematics*, 2006, pp. 1433–1452.

[5] D. L. Donoho, "Compressed sensing," *IEEE Transactions on Information Theory*, vol. 52, no. 4, pp. 1289–1306, 2006.

[6] P. T. Boufounos and R. G. Baraniuk, "1-bit compressive sensing," in *Proc. Conf. Inform. Science and Systems (CISS)*, Princeton, NJ, 2008.

[7] A. Zymnis, S. Boyd, and E. Candès, "Compressed sensing with quantized measurements," *IEEE Signal Processing Letters*, vol. 17, no. 2, pp. 149–152, 2010.

[8] R. T. Rockafellar and R. J.-B. Wets, *Variational Analysis*. Berlin, Heidelberg: Springer-Verlag, 1998.

[9] A. Chambolle and T. Pock, "A first-order primal-dual algorithm for convex problems with applications to imaging," *Journal of Mathematical Imaging and Vision*, vol. 40, pp. 120–145, 2011.

[10] P. C. Loizou, *Speech Enhancement - Theory and Practice*. Boca Raton, FL, USA: CRC Press, Taylor & Francis Group, 2007.