Sheet 1

Problem 1.

What is the probability to win in the German lottery? In that lottery the player is allowed to choose 6 numbers from 49 numbers. The player wins if the choice matches a random choice.

Problem 2.

A candidate of a TV show has to choose among 3 doors and then wins the price behind the chosen door. Behind one door is a very expensive car, behind the two other doors is a goat each. After the candidate has chosen one door (say A), the host opens one of the remaining doors (say B). Of course, he picks a door with a goat behind it (at least one of the goat is left, and the host is free to choose any of the remaining doors). Now the candidate is allowed to choose again between A and C. Is the probability to win the car less, higher, or the same if the candidate changes his mind and chooses now C instead of A? Prove your claim.

Problem 3.

A group of n men enter a restaurant and check their hats. The hat-checker is absent minded, and upon leaving, she redistributes the hats back to the men at random.

1. What is the probability P_n that no man gets his correct hat, and how does P_n behave as n approaches infinity?

2. What is the expected number of men that get their correct hat?

Problem 4.

Consider the process of randomly tossing n identical balls into n bins.

1. What is the expected number of balls in the first bin?

2. What is the expected number of bins receiving no ball at all?

Problem 5.

We roll a standard fair die over and over. What is the expected number of rolls until the first pair of consecutive sixes appears? (Hint: The answer is not 36.)