BKA
Sheet 3

Due date: 18 May

Exercise 1.
Prove that a language $C \subseteq \{0,1\}^*$ is Turing-recognizable if and only if there is a decidable language D such that $C = \{x \in \{0,1\}^* \mid \exists y \in \{0,1\}^* : (\langle x, y \rangle \in D)\}$.

Hint: What kind of "advice" or "hint" would you need so that you could use a recognizer for C to *decide* whether or not a string x is in C? This advice is y.

Exercise 2.
Let D be the set of all encodings of TMs such that the encoded TM is a decider, and fix some enumeration $\{\langle M_1 \rangle, \langle M_2 \rangle, \ldots\}$ of D. Let $U = \{i \mid i \notin L(\langle M_i \rangle)\}$.

(a) Prove that U is undecidable. (Hint: Diagonalization)

(b) Why is the following proof not correct?
Consider the following TM M:

- Run M_i on input $\langle i \rangle$
- If M_i accepts, reject.
- If M_i rejects, accept.

M always halts, since all M_i's are deciders by definition. Thus, by construction, M decides U. Therefore, U is decidable!

Exercise 3.
Show a language is decidable iff some nondeterministic Turing machine decides it. (You may use the proof of theorem 3.16 and use it over a tree. If every node in tree has finitely many children and every branch of the tree has finitely many nodes, the tree itself has finitely many nodes.)