BKA
Sheet 4
Due date: 25 May

Exercise 1.
For a set S, let $\mathbb{P}(S)$ denote the set of all subsets of S. Also A and B are two sets. Justify your answers.

(a) Is $\mathbb{P}(\mathbb{N})$ countable?

(b) A and B are countable. Is $A \times B$ countable?

(c) $A \subset B$ and B is countable. Is A countable?

Definition 1. *Let A be a language. The operation star (*) is defined as follow:*
$A^* = \{x_1 x_2 \dots x_k | k \geq 0 \text{ and each } x_i \in A\}$

Exercise 2.
Prove that the

(a) Turing decidable languages

(b) Turing recognizable languages

are closed under *.

Exercise 3.
Let Σ be an input alphabet and Γ be a (suitable) band alphabet. Let furthermore $\mathcal{M} = \{\langle M \rangle | M \text{ is a TM with input alphabet } \Sigma \text{ and band alphabet } \Gamma\}$. From the book we know that $\mathcal{M}$ is countable. Therefore there exists a bijective function $g : \mathcal{M} \to \mathbb{N}$ from the Turing machines to the natural numbers. Show that the function $p : \mathcal{M} \to \mathbb{N}$ with $p(\langle M \rangle) = \min\{g(\langle M' \rangle) | \langle M' \rangle \in \mathcal{M} \text{ and } L(M) = L(M')\}$ is not computable.