BKA
Sheet 5

Due date: 08 June

Exercise 1.

(a) Debate the correctness of the following statements.

(i) $\log(n!) = \mathcal{O}(n \log n)$

(ii) $(\log n)^{\frac{2\log n}{\log\log n}} = \mathcal{O}((\log n)^2)$

(b) Sort the following functions in run time decreasingly.

(i)

Function	f_1	f_2	f_3	f_4	f_5	f_6
Run time	$\mathcal{O}(3^n)$	$\mathcal{O}(n^3)$	$\mathcal{O}(\log(n!))$	$\mathcal{O}(\log n)$	$\mathcal{O}(2^{2n})$	$\mathcal{O}(n^{1/\log n})$

(ii)

Function	g_1	g_2	g_3	g_4	g_5	g_6
Run time	$\mathcal{O}(3^{\log n})$	$\mathcal{O}(e^n)$	$\mathcal{O}(4^{\log n})$	$\mathcal{O}((\log n)^{\log n})$	$\mathcal{O}((n+1)!)$	$\mathcal{O}(\sqrt[3]{\log n})$

Exercise 2.
Show that P is closed under union, concentration and complement.

Exercise 3.
A **triangle** in an undirected graph is a 3-clique. Show that $TRIANGLE \in P$, where TRIANGLE $= \{< G > |$G contains a triangle$\}$.

Exercise 4.
A $2cnf - formula$ is an AND of clauses, where each clause is an OR of at most two literals. Let $2SAT = \{< \phi > | \phi$ is a satisfyable 2cnf-formula$\}$. Show that $SAT \in P$.

Exercise 5.
Prove that 3-SAT is $\mathcal{NP}$-complete.

Exercise 6.
Let $3CNF$ be the set of all satisfiable propositional formulas in CNF with at most 3 literals per clause. Consider now the set $N3CNF$ of all formulas in $3CNF$ in which neither clause contains 3 positive or 3 negative literals. Show that $N3CNF$ is $\mathcal{NP}$-complete.
Hint: Use the $3CNF$ problem for your reduction.

Exercise 7.
Let $HALF - ClIQUE = \{< G > \mid$ G is an undirected graph having a complete subgraph with at least m/2 noeds, where m is the number of nodes in G $\}$. Show that $HALF - CLIQUE$ is $NP-$complete.
Hint: Use the Clique problem for your reduction.