BKA
Sheet 6

Due date: 22 June

Exercise 1.
We know the problem of finding a Hamiltonian cycle in a graph is $\mathcal{NP}$-complete.

a. Prove finding a Hamiltonian path (between two given vertices) in a graph is $\mathcal{NP}$-complete.

b. Prove finding a cycle with sum of (edges') weights equals to zero in a weighted graph is $\mathcal{NP}$-complete. (A weighted graph is a graph where each edge has a numerical value called weight.)

Exercise 2.
In scheduling problem, n jobs with processing time t_i, releasing time r_i and due time d_i are given. The objective is to distinguish whether it is possible to find a schedule of jobs on one machine such that each job j_i (exclusively) receives the machine in a continues time interval t_i in period $[r_i, d_i]$. Show this problem is $\mathcal{NP}$-complete.
(Hint: use the problem of Subset-sum)

Exercise 3.
The subgraph isomorphism problem is defined as follow. Two graphs H and G are given and we are to find out whether the graph G contains a subgraph that is isomorphic to H. Show this problem is $\mathcal{NP}$-complete. (This problem is a generalization of both the maximum clique problem and the problem of testing whether a graph has a Hamiltonian cycle.)

Exercise 4.
Show the ensuing problem is $\mathcal{NP}$-complete.
An undirected graph $G = (V, E)$ and an integer number k are given. Determine whether G has a minimum spanning tree T such that the degree of any vertex in it is at most k.
(Hint: you may use the Hamiltonian path problem!)

Exercise 5.
Perhaps the first strategy one tries when designing an algorithm for an optimization problem is the greedy strategy. For the vertex cover problem, this would involve iteratively picking a maximum degree vertex and removing it, together with edges incident to it, until there is no edge left. Show that this algorithm achieves an approximation of $\mathcal{O}(\log n)$ such that n is the number of vertices in given graph. Give a tight example of this algorithm.