BKA
Sheet 7

Due date: 29 June

Definition 1 (Maximum-Cover). *Set S and set $M = \{S_1 | S_i \subset S, i \leq m\}$ are given. The objective is to choose k elements out of M such that the number of covered elements is maximal. This problem is NP-Complete.*

Consider the two following algorithms for approximating the Maximum-Cover problem.

Exercise 1.
Initially, arbitrarily choose a subset of M with size k. Replace one set out this subset with an element of M such that the number of covered elements is increased. Repeat until there is no more change.
Show that this gives a 2-approximation of the optimal solution.

Exercise 2.
Start with an empty set. Repeat until there is no more change: greedily add an element to the set such that the number of covered elements of S is maximized.
Show that this gives a $\frac{e}{e-1}$-approximation of the optimal solution.

Definition 2 (Vertex-Coloring Problem). *Given an undirected graph $G = (V, E)$ with $|V| = n$, color its vertices with the minimum number of colors so that the two endpoints of each edge receive distinct colors.*

Exercise 3.
Give a greedy algorithm for coloring G with $\Delta + 1$ colors, where Δ is the maximum degree of a vertex.

Exercise 4.
Give an algorithm for coloring a 3-colorable graph with $\mathcal{O}(\sqrt{n})$ colors. (Hint: For any vertex v, the induced subgraph on its neighbors, $N(v)$, is bipartite, and hence optimally colorable. If v has degree more than $\sqrt{n}$, color $v \cup N(v)$ using 3 distinct colors. Continue until every vertex has degree less equal to $\sqrt{n}$. Then use your algorithm of the last question!)