Assignment Sheet 2

Petra Berenbrink   Dominik Kaaser

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Exercise 4. *(from last sheet)*
In the lecture, we proved the standard Chernoff bound for the sum $X = \sum_{i=1}^{n} X_i$ of n independent 0-1 random variables X_i. That bound requires the exact expectation $\mu \coloneqq \mathbb{E}[X]$, which is often not available. Prove the following generalized Chernoff bound which holds for an arbitrary upper bound $\mu_H \geq \mu$ on the expectation:

$$\Pr[X \geq (1+\delta) \cdot \mu_H] \leq \left(\frac{e^{\delta}}{(1+\delta)^{1+\delta}} \right)^{\mu_H}.$$

Exercise 6. *(from last sheet)*
The HVV Transportation Referendum is a poll that asks Hamburg's population whether they support an increased ticket price to extend the transportation system. Let p denote the actual (currently unknown) fraction of people who support this referendum. Our goal is to get an estimate X of p that is within $\phi \in (0,1)$ of the true value with a sufficiently high probability. That is, we want

$$\Pr[|X - p| \leq \phi] > 1 - \varepsilon,$$

where $\phi, \varepsilon \in (0,1)$ are small constants. We compute the estimate X by choosing N people from Hamburg uniformly at random and set X to the fraction of these people that support the referendum. How large should we choose N in order to guarantee the above error probability? Use Chernoff bounds to find an expression for N (in terms of ϕ and ε). How large should we choose N in order to guarantee that X is with 95% probability correct up to $\pm 2\%$?

Exercise 7.
Consider the deterministic bit-fixing routing algorithm on the n-dimensional hypercube and suppose n is even. The transpose permutation $\pi\colon \{0,1\}^n \to \{0,1\}^n$ maps a bit string $ab \in \{0,1\}^n$, with $a, b \in \{0,1\}^{n/2}$ to $(b,a) \in \{0,1\}^n$. Show that the bit-fixing algorithm needs $\Omega(2^{n/2})$ steps to route π.

Exercise 8.

Consider the following modification to the bit-fixing routing algorithm for routing a permutation on the hypercube. Suppose that, instead of fixing the bits in order from 1 to n, each packet chooses a random order, independently, and fixes the bits in that order. Show that there exists a permutation for which this algorithm requires $2^{\Omega(n)}$ steps with high probability.

Exercise 9.

Theorem 4.7 [MR95] for the randomized packet routing algorithm states that with probability at least $1 - 1/N$, every packet reaches its destination in $14n$ or fewer steps. Show that the expected number of steps within which all packets are delivered is less than $15n$.