Assignment Sheet 6

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January 2, 2017

Exercise 19.
Consider the two-state Markov chain with the following transition matrix.

$$P = \begin{pmatrix} p & 1-p \\ 1-p & p \end{pmatrix}$$

Find a simple expression for $P^t_{0,0}$.

Exercise 20.
Consider a process $X_0, X_1, X_2, ...$ with two states. The process is governed by two matrices, P and Q. If the time k is even, the transition probabilities are P. Otherwise, they are Q. Explain why this process does not satisfy the definition of a time-homogeneous Markov chain. Give an equivalent process (with a larger state space) that satisfies the definition.

Exercise 21.
Prove that the communicating relation defines an equivalence relation.

Exercise 22.
Prove that if one state in a communicating class is transient (respectively, recurrent) then all states in that class are transient (respectively, recurrent).

Exercise 23.
An $n \times n$ matrix P is called stochastic if all entries are nonnegative and the sum of each row is 1. It is called doubly stochastic if, additionally, the sum of the entries in each column is 1. Show that the uniform distribution is a stationary distribution for any Markov chain represented by a doubly stochastic matrix.