Randomised Approximation

Sheet 1

Due date: 17.11.2020

Exercise 1.
We have a six-sided dice which comes to side i with probability proportional to i. What is the probability of having even number in one roll?

Exercise 2.
Suppose that a fair coin is flipped n times. For $k > 0$, find an upper bound on the probability that there is a sequence of $\log_2 n + k$ consecutive heads.

Exercise 3.
A group of n men enter a restaurant and check their hats. The hat-checker is absent minded, and upon leaving, she redistributes the hats back to the men at random.

(a) What is the probability P_n that no man gets his correct hat, and how does P_n behave as n approaches infinity?

(b) What is the expected number of men that get the correct hat?

Exercise 4.
Give examples of events where $\Pr(A|B) < \Pr(A)$, $\Pr(A|B) = \Pr(A)$ and $\Pr(A|B) > \Pr(A)$.

Exercise 5.
Suppose that we flip a fair coin n times to obtain n random bits. Consider all $m = \binom{n}{2}$ pairs of these bits in some order. Let Y_i be the exclusive-or of the ith pair of bits, and let $Y = \sum_{i=1}^{m} Y_i$ be the number of Y_i that equal 1.

- Show that each Y_i is 0 with probability 1/2 and 1 with probability 1/2.
- Show that the Y_i are not mutually independent.
- Show that the Y_i satisfy the property that $E[Y_iY_j] = E[Y_i]E[Y_j]$.
- Find $Var[Y]$.

Exercise 6.
Prove that $E[X^k] \geq E[X]^k$ for any even integer $k \geq 1$.

Exercise 7.
We roll a standard fair die over and over. What is the expected number of rolls until the first pair of consecutive sixes appears? (Hint: The answer is not 36.)

If you have any question regarding the problems, please do not hesitate to contact us.