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APPLICATION AND VISUALISATION OF FINANCIAL RISK METRICS IN DISCRETE EVENT SIMULATION
– Concepts and Implementation –

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ABSTRACT
We propose a conceptual procedure to integrate methods of specific application domains into general purpose discrete event simulation. This procedure is applied on quantitative finance, a field that deals with computer-assisted analysis of asset prices and supports investment decisions in financial markets. One of quantitative finance's most significant assessment categories relates to the term *risk*. We generalise the concept of risk, regarding risk as observation of undesired state dynamics of a modelled system. We outline various interpretations and application fields for this generalised risk notion, demonstrating its wide scope. Special attention is paid to risk types and reference states of risk. We report in detail on implementation of four established risk metrics into our discrete event simulation framework DESMO-J. Special focus is given to realised conceptual extensions and risk metric visualisation. We point out further development options and conclude that this approach may benefit and inspire a wide range of application fields.

Keywords: risk metrics, discrete event simulation, generalisation, visualisation

1. INTRODUCTION: METHOD TRANSFER TO DISCRETE EVENT SIMULATION
Scientific disciplines like physics, engineering, the social sciences or finance have developed a variety of methods to process or visualise experimental data. Simulation serves as a means to conduct experiments in these and several other subject areas as well. Unless domain specific simulators are employed, general purpose discrete event simulators often only deploy standard methods like descriptive statistics or random number generation, as a greatest common factor among disciplines. The decision for not implementing powerful methods of single application domains is often based on their discipline specificity, leading to inapplicability in other domains.

As an example, risk metrics from finance concentrate on expressing the observed or potential capital loss of a portfolio or trading strategy within a certain time horizon. In this narrow sense, the concepts are too constrained for e.g. biological ecosystem analysis, which may be interested in the potential of preserving certain quality conditions in an ecosystem.

We argue that generalising and transferring methods from simulation application domains in such a way that they are applicable in simulation studies of other areas as well may be beneficial and possibly inspiring in research.

The conceptual procedure we propose to integrate methods of specific application domains into general purpose discrete event simulation is shown in figure 1.

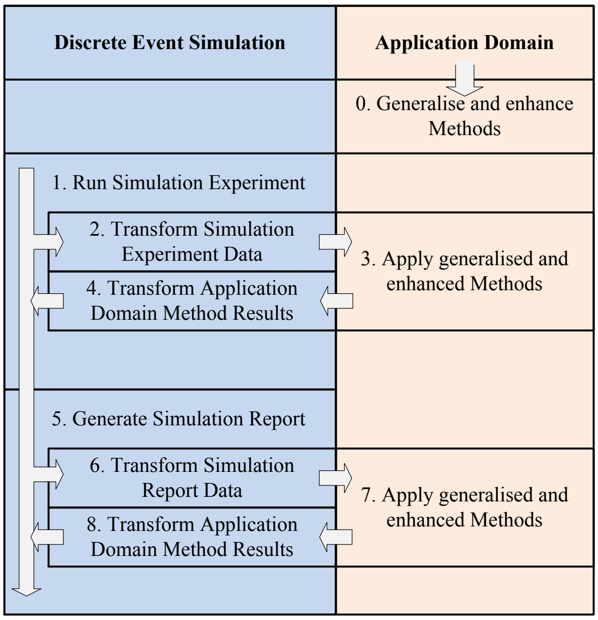

Figure 1: Generalisation and Utilisation of Application Domain Methods in Discrete Event Simulation

In a one-time preliminary step methods of a specific application domain are generalised and – if necessary – enhanced, with the objective of general usability in various other discrete event simulation application areas. The generalised methods are implemented within the simulation package, e.g. as additional software classes. They may be called both during execution of simulation experiments and at generation of simulation reports. Depending on the environment in which the domain methods have been used before, a transformation of simulation experiment or report data may have to precede their call, in order to prepare a proper application context. Analogously, a post-processing of the domain methods' results may be necessary, in order to incorporate them in the further course of simulation. It is subject to the modeller and is depending on the application domain which methods are utilised during simulation experiments, at simulation report generation or on both occasions.

In this way, additional powerful functionality originating from specific application domains may be provided in discrete event simulation packages, for the benefit of a further range of subject areas.

This paper is laid out as follows: Section 2 introduces quantitative finance and risk metrics. Risk types are generalised in subsection 2.1, whereas the aspect of risk reference states is discussed in subsection 2.2. The main section 3 and its subsections report on the implementation and visualisation of four quantitative finance risk metrics in our open source discrete event simulation framework DESMO-J; namely semivariance, Value at Risk, Expected Shortfall and drawdown. Generalisation, extensions, application proposals, presentation and visualisation in simulation report as well as interpretation are discussed. Section 4 concludes this paper. It outlines the current status of work and gives an outlook on further development options.

2. QUANTITATIVE FINANCE RISK METRICS

The field of Quantitative Finance (also called *Computational Finance* or *Financial Engineering*) deals with computer-assisted analysis of capital asset prices and the support of investment decisions in financial markets.

Here, specialised discrete event simulators called *back testers* are utilised, in order to evaluate financial market trading strategies: Strategies under examination are simulated in different historical or hypothetical market environments and are evaluated, compared and optimised by means of a wide range of assessment criteria. In this context, one of the most significant assessment categories relates to the *risk* taken in following a particular trading strategy.

Historically, risk first was defined as variance of expected return with regard to a portfolio of assets (Markowitz 1952). Later, quantitative finance distinguished the concepts of *uncertainty* and risk and elaborated asymmetrical risk metrics called *downside risk*. Under this aspect, only negative deviations of return or asset value are considered, as finance assesses underperformance as undesirable, whereas unexpected profit is welcome (Markowitz 1959 pp.193–194, Sortino and van der Meer 1991, Harlow 1991, Rom and Ferguson 1993, Rachev et al. 2008).

In the context of back testers, a risk *metric* is a concept to assess risk, both serving for quantification of risk of a particular trading strategy and for comparison of different trading strategies amongst each other. The term risk *measure* refers to the computational implementation of calculating a particular risk metric.

To us, risk metrics appear useful for application in other domains beyond finance as well. In a preceding WAMS paper, we proposed generalisation of the four most established risk metrics in quantitative finance in order to utilise them in discrete event simulation (Koors and Page 2012). Here, we report on refinement, further development, implementation and visualisation of these metrics. We think that their integration into discrete event simulators opens up new and fruitful views on model dynamics in general and may specifically support evaluation and possibly optimisation of model behaviour.

In the remainder of this section, we generalise the quantitative finance concept of risk in terms of type and reference state, in order to widen its application area to new domains (cf. figure 1, step 0).

2.1. Generalising Risk Type

Many domains differentiate between welcome and undesired dynamics of a system, depending on the particular task at hand:

a) Referring to financial time series, quantitative finance would term a collapse in the equity curve of a trading system *Downside Risk* (figure 2a).

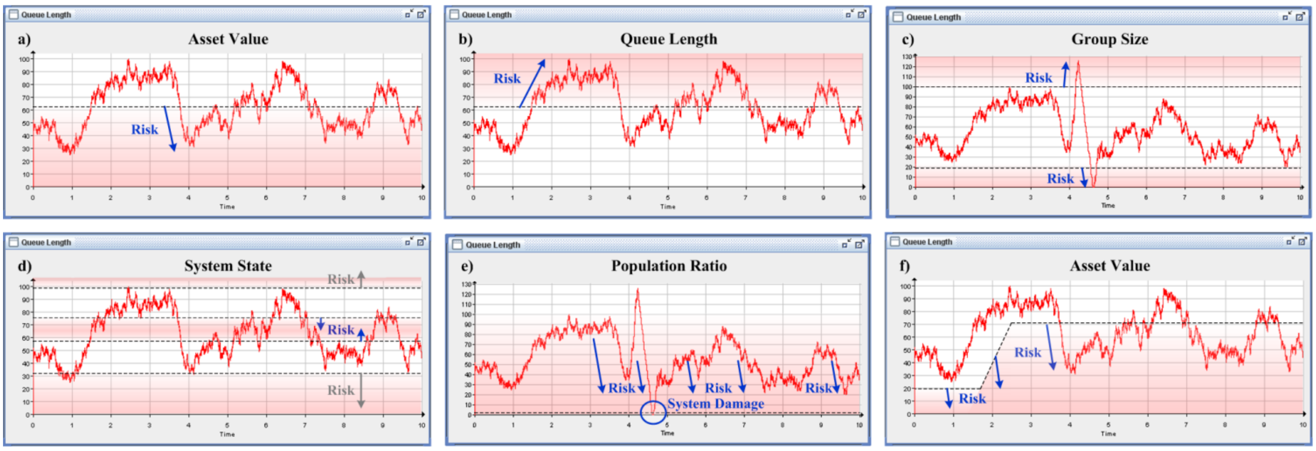

Figure 2: Different Types of Risk

In order to support a wider range of application fields, we extend the scope of the concept *risk* and its associated methods as follows:

In a generalised context, risk can be regarded as observation of undesired state dynamics of a modelled system.

b) In the engineering context of factory planning, waiting queues in front of new machines have to be dimensioned. In accompanying simulation experiments, according queue lengths will be observed and can be recorded. Undesired dynamics occur when queue lengths tend to approach their maximum capacity, meaning upstream machines will have to stop or client orders will be lost (figure 2b). From an abstract point of view, downside risk metrics used in quantitative finance to evaluate decreases in price time series are similarly applicable to evaluate increases in queue length time series. Formulae and algorithms only have to be adapted slightly and then can be correspondingly employed as valuation standards for *Upside Risk* in general purpose discrete event simulation, to support various other application domains.

c) When simulating social systems, there might be a preferable bandwidth of agent group sizes in order to retain stable groups. When group size shrinks too much, a group might dissolve, whereas an ever-growing group might lose self-identification and split. Here, *Outside Risk* can be attributed to undesirably departing from an interval of system preserving states (figure 2c). For this application case, the risk metrics already generalised in cases a) and b) merely have to be merged.

d) In bi-stable or multi-stable technical or social systems, stable states resp. state regions can be considered “safe”, whereas state transitions can effect wear, damage or destruction. For example, switching of gears imposes additional strains on transmissions; or in countries with two party systems, change of government might induce higher social tension than retention of the ruling administration. In this respect, regions between or beyond stable states can be considered undesired and hence risky (figure 2d). More general, the preference to avoid certain states or state transition regions results in several (un)desired partitions of the state spectrum. This view of *Transition Risk* can be dealt with by combining multiple risk metric instances of type c).

e) In the analysis of biological ecosystems, it may be desirable to maintain a certain ratio of different species’ population sizes, i.e. population proportions should stay within an equilibrium state (similar to figure 2c). If the population of one species stagnates, whereas the habitat of another species grows, it may be generally acceptable that the population ratio decreases to a small positive figure. However, reaching the extreme target state 0, would mean that one species is extinct. In this context, the term *Critical State Risk* denotes approximation to critical system states that threaten to damage or destroy a system (figure 2e).

f) The risk concepts identified so far are time-independent and path-independent, i.e. the region of critical states remains fixed during an experiment. From a different financial perspective, risk might be perceived as loss of asset value already gained before (figure 2f). In this conception, state regions considered risky depend on the states that have been traversed so far. For example, the term *undesired dynamics* may refer to losing more than 10% of the maximum portfolio value registered so far. As portfolio value grows, the boundary level of states regarded risky rises proportionally. Thus, states regarded welcome in the past may be considered undesired in the future, depending on time and state transitions in the course of a system’s dynamic development. This path-dependent *Drawdown Risk* perception concerns processes that are not in a stationary phase but exhibit behaviour of continuous growth or decline. As an inverse example in medical simulation, a decreasing infection rate of an epidemic is favourable and every new minimum m_t of infection rate will be welcome. But when infection rate declines further and new minimums m_{t+n} are reached, any increase of infections back to a then-past minimum m_t now will be considered as undesired and risky.

A common characteristic of the examples a) to f) given above is the need to evaluate modelled systems or strategies asymmetrically, distinguishing welcome and undesired state regions. In contrast, standard descriptive simulation statistics focus on symmetric non-distinguishing concepts like mean and variance. On the background of asymmetric evaluation preferences, it appears that generalised risk metrics inspired by quantitative finance may provide additional value for risk and dynamics assessment in discrete event simulation application fields, complementing conventional standard statistics.

In summary, generalising risk type means extending quantitative finance instruments designed for downside risk (a) and drawdown risk (f) by expanding applicability to upward dynamics (type b), range scenarios (type c), transitions in multi-stable contexts (type d) or risky target states (type e). On this basis, tools for modellers of diverse simulation application domains regarding risk as undesired dynamics can be provided. In section 3, we will report on four prominent risk metrics that have been amended to cover risk types a), b), c) and f).

2.2. Generalising Risk Reference State

In quantitative finance, the concept of risk relates to a certain reference state (return on investment or portfolio value), considered as starting point to assess risk from. Depending on the particular risk metric, this is either the arithmetic mean of observations or the last observed state within a series of observations.

In this regard, risk metrics referring to the mean of observed values usually describe the character of risk as deviation from the mean in the past. Here, mean is considered as a representative that is typical or desirable for the set of observed states. Epistemological interest focusses on evaluation and explanation – possibly also ex post optimisation – of a system's past state history.

When referring to the last observed state, future risk impending in the current situation is of interest. Here, past state history is regarded as well; however the emphasis is on using past data not to describe the past but rather to determine a forecast of deviation from the current state for the near future. In this case, epistemological interest concentrates on prognosis of future developments and adaption of strategy design.

We argue that the main purpose of employing discrete event simulation in various application domains is evaluation and analysis of existing (or planned) systems. Thus, we focus on using quantitative finance methods as valuation standards for past observations here, as opposed to utilising them for forward-looking operational decision support. Consequently, risk metrics are employed for assessment of recorded state dynamics at end of simulation experiments, as depicted in figure 1, step 7.

As aforementioned, backward-looking risk metrics typically use the mean of observed states in further computations. Considering that manifold application areas may prefer other reference states in assessment of risk, it is necessary to generalise risk metrics with regard to a wider choice of reference states.

For this reason, additional characteristic reference states in observation time series are identified as follows:

a) *Median* state. The median is less sensitive to statistical outliers at the ends of state spectrum than the mean of states. Additionally, the median is (near) a state that actually was observed, whereas the mean of states as a calculated artefact need not necessarily correspond to a practically observable state. In an analytical context, using the mean might facilitate handling of equations, compared to the median. But against a computational background, algorithms remain the same, regardless of which value is stored in a variable for the reference state. On these grounds, it is recommended to prefer the median to the mean of states.

b) *Mode* states (Most frequent states, Modal states). Referring to mode states may be more valid than concerning the mean of states or the median, since the most frequent states may appear more characteristic than any other states that were observed with lower frequencies. Furthermore, risk metrics referring to the mode states benefit from a greater statistical population of starting points and thus more cases observed than in any other case, implying higher significance. At last, mode states are virtually insensitive to outliers. For these reasons, it is generally advised to include mode states in risk analysis.

c) *Minimum* and *Maximum* state. Referring to these states allows studying model dynamics from the perspective of extreme states resp. environments. Depending on the particular system examined, *extreme* may relate to especially welcome or especially undesirable states: The interpretation of extreme states as preferable or avoidable remains with the modeller. For example, the (frequent, but in terms of state) "extreme" state of an empty waiting queue might appear desirable, whereas the extreme state of highest observed queue length may be regarded undesirable. Additionally, it should be noted that "risk" with reference to undesired states can mean departing from unwanted extreme states and heading for more welcome states. Therefore, risk metrics referencing undesired states may actually characterise recovery phases towards welcome states.

d) *Previous Maximum* or *Previous Minimum* state. In cases of non-stationary processes (cf. application scenario in figure 2f) it can make sense to refer to the preceding extreme state, looking back from present simulation time. By the nature of an ever-increasing or ever-decreasing process, current extreme states will only have a temporary relevance. By considering the sequence of increasing or decreasing extreme states as an artificial reference state series, risk can be interpreted and quantified as the extent of temporary state movements into an undesired opposite direction, against the main trend. In this context, only the relative degree of counter movement is of interest, whereas the absolute states traversed on these occasions lose importance.

Extreme states typically have a very low frequency; sometimes only one observation of maximum or minimum is made. In consequence, samples of extreme state observations may become too small to allow valid conclusions based on risk metrics. This circumstance is met by introducing *State Regions* around reference states, with an adjustable radius r. A radius $r \geq 0$ around a reference state s incorporates all states in $[s - r, s + r]$ into the risk metrics of reference state s. By this means, enlarged reference state sets can be dimensioned, to obtain sufficient states for significant risk metric results. Risk metrics referring to state regions are re-defined to calculate results for each state occurrence t in $[s - r, s + r]$. If desired, all states $t \neq s$ in a state region may be set to the same reference state s before computation.

Another issue arises when regarding (potentially risky) dynamics of state transitions. Given a start time t_0 and a fixed state transition time Δt, it cannot be guaranteed that an observation variable will change its state precisely at $t_0 + \Delta t$. So it is not exactly defined which target state should be assigned to a start state after Δt. The path taken here is to provide a parameter to the modeller for choosing the outcome of a) the preceding b) the following or c) the nearest state change event to $t_0 + \Delta t$ as *Flexible Target State* of a state transition. The default is to consider the last state change event preceding $t_0 + \Delta t$, since it is assumed that a preceding state is valid until it is superseded by a following state change event later than $t_0 + \Delta t$: the target state observed at the last state change event before $t_0 + \Delta t$ is regarded to be still in effect.

It is obvious that risk assessment highly depends on the state considered as reference state for normality. If a highly frequent reference state in the centre of the state spectrum is chosen, a large number of relatively small state differences may be observed, presumably with similar extent of deviations in both directions, above and below the reference state. In contrast, choosing an extreme state with a low frequency might imply higher risk for the same state observation series, since recorded state differences are greater. Thus, the same risk metric can lead to different results and conclusions for a given system, depending on the choice of reference state. It is advised to select reference states carefully, considering interpretation and relationships of equivalent states in the original modelled system in terms of undesirable dynamics (i.e. risk).

By extension of financial risk metrics to further risk types as described in subsection 2.1, and introduction of six additional reference states, state regions and flexible target states (subsection 2.2), a range of new application scenarios opens up, providing several application fields of discrete event simulation with additional valuation standards. Since assessment of risk types and choice of risk reference states are flexible according to modellers' needs, accurate domain-specific risk definition and interpretation is made available, for gaining more insight and transparency of a system's state dynamics.

3. IMPLEMENTATION AND VISUALISATION OF GENERALISED RISK METRICS

This section presents four established quantitative finance risk metrics – namely semivariance, Value at Risk, Expected Shortfall and drawdown – and describes their concrete generalisation for use in discrete event simulation. Every risk metric is discussed in an own subsection, with definition, application proposals and elaboration of generalisation and extensions. Further attention is given to textual presentation resp. graphical visualisation in simulation reports, followed by an interpretation in the application context of a simple queuing system. The focus of this section is on conceptual enhancements, further development and experiences gained, carrying on the more conceptual introduction by Koors and Page (2012).

Computation and visualisation of the generalised risk metrics have been implemented in Java into our open source discrete event simulation framework DESMO-J (www.desmoj.de; Page 2013, Göbel et al. 2013, Page and Kreutzer 2005), in a bachelor's thesis (Göttsch 2013) and in a bachelor project (Peltzer 2013), at the University of Hamburg.

Formal definitions of the original risk metrics in their financial context can be found in e.g. Yang, Yu, and Zhang (2009); Lohre, Neumann, and Winterfeldt (2009) or Giorgi (2002).

3.1. Semivariance

(Below mean) semivariance SV_{-} is defined like standard variance V, with the only exception that variations above the mean are made effectless by setting them equal to the observed mean. As a result, semivariance measures only deviations below the mean.

Above mean semivariance SV_{+} is computed analogously, restricting measurement to variations above the mean. The sum of both semivariances – below and above mean – amounts to conventional standard variance: $SV_{-} + SV_{+} = V$.

Below mean semideviation SD_{-} and above mean semideviation SD_{+} are defined as the square root of below mean semivariance resp. above mean semivariance. Note that their sum usually does not amount to standard deviation SD, because $\sqrt{SV_{-}} + \sqrt{SV_{+}}$ usually is not equal to $\sqrt{SV_{-} + SV_{+}} = \sqrt{V} = SD$.

Markowitz (1959, pp.193–194) was the first to mention (below mean) semivariance as an alternative to standard variance for financial risk measurement, but did not follow up this approach due to computational restrictions at that time. Later, Hogan and Warren (1974) and subsequent authors proposed an asymmetric concept like semivariance to better account for asymmetric risk perception of investors. A result of the following discourse was to consider standard symmetric variance as representative of uncertainty, whereas asymmetric concepts like semivariance were regarded as instances of risk in the narrow sense.

3.1.1. Application

We propose to apply semivariance in discrete event simulation for assessment of steady state phases. In consistent steady state phases, overall state fluctuation below and above the mean should be of similar character. Therefore below mean and above mean semivariance should be of comparable magnitude. If a significant difference of the two semivariances is observed, unsymmetrical variation around the mean may be present and should be clarified by further examination.

3.1.2. Presentation in Simulation Report

In DESMO-J, Semivariance statistics can be declared and computed for any recording (time series) of observed state variable values. At the end of a simulation run, semivariance is presented textually in the simulation report as shown in figure 3.

Semivariance

Title	Recording	Mean	Standard Deviation	Below mean Deviation	Above mean Deviation	Variance	Below mean Semivariance	Above mean Semivariance
Queue Length	Queue Length Values	63.3143	17.94843	17.42687	18.48954	322.1462	156.52117	165.62503

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Figure 3: Textual Presentation of Semivariances and Deviations in Simulation Report

Apart from below mean and above mean semivariance, the mean, standard deviation and variance are listed as well. The columns for Below Mean Deviation and Above Mean Deviation are discussed later in subsection 3.1.3; they are not identical to semideviation.

If the report format is set to HTML with graphics, semivariances are also displayed graphically, as in figure 4. The graphical representation facilitates a quick visual comparison of variation dimensions below and above the mean.

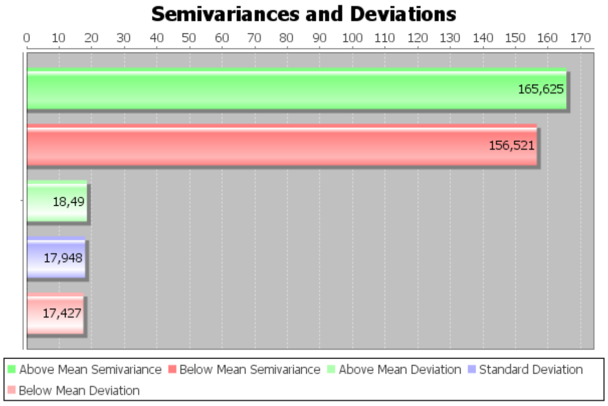

Figure 4: Graphical Visualisation of Semivariances and Deviations in Simulation Report

3.1.3. Extension

Although relative comparison of the two semivariances is expedient, the absolute semivariance values are not interpretable in relation to elementary observations, because semivariances are sums of squared observations with a squared dimension.

Semideviation does not lead further, because it has no intuitive relationship to standard deviation (see subsection 3.1 above). Even in a perfect symmetric fluctuation of states around the mean, semideviations below mean and above mean are $1/\sqrt{2}$ times lower than the standard deviation.

We propose to compute metrics *Above Mean Deviation* and *Below Mean Deviation* similar to semideviation, but to totally ignore observations below (resp. above) the mean, instead of incorporating them with value 0. The difference is that here the sum of squares is divided by the number of non-ignored observations (minus 1), instead of the total number of observations (minus 1) in case of semideviation. By this definition, below mean deviation and above mean deviation are nearly identical to standard deviation, if fluctuation around the mean is symmetric (cf. figure 4).

This opens up new room for interpretation: We graphically superimpose observation time series with state bands with width of above mean and below mean deviation (figure 5). Thereby, the experimenter can get an impression of variation range in terms of deviations. Untypical dynamics are more easily identifiable compared to using standard deviation, because above mean and below mean deviations may have different values. For example, an absolute deviation d may be normal when observed above the mean (because it is e.g. within two above mean deviations), but exceptional below the mean (because there, it may be e.g. beyond three below mean deviations).

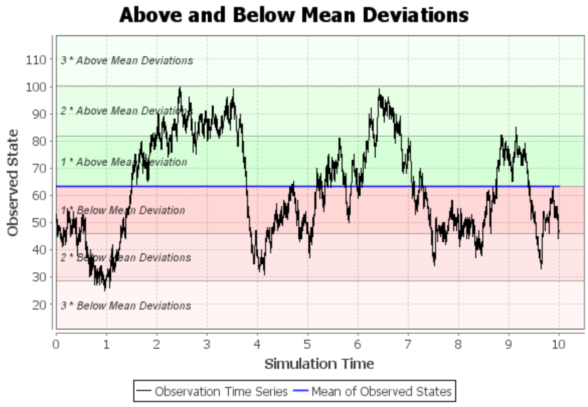

Figure 5: Graphical Visualisation of Deviations as State Bands in Simulation Report

As an interpretation example, it can be concluded from figure 4 that the underlying queuing system has a quite balanced variation below and above the mean. Figure 5 shows that model dynamics remains within two deviations above and below the mean queue length, without unilateral exaggerations. However, it seems that the mean state itself is observed rather infrequently, which questions its eligibility as main reference state of model dynamics (compare subsection 2.2).

3.2. Value at Risk and Expected Shortfall

The risk metric Value at Risk (VaR) quantifies the maximum financial loss that will not be exceeded at a given confidence level of $1 - \alpha$, at the end of a set period (J. P. Morgan 1996, Hull and White 1998). To put it another way, VaR is equivalent to the α-quantile of the probability distribution of returns in the set period.

Figure 6 depicts a potential distribution of returns after a set period as well as the α-quantile (VaR) of this distribution.

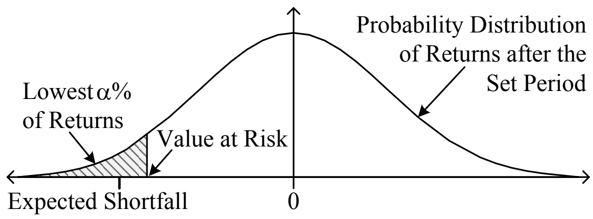

Figure 6: Illustration of Value at Risk and Expected Shortfall

The related risk metric Expected Shortfall (or Conditional Value at Risk, CVaR) expresses the expected amount of loss for the α fraction of cases where VaR is exceeded (Rockafellar and Uryasev 2000). In other words, CVaR is equivalent to the expected value of all observations below the α-quantile of the distribution (cf. figure 6).

Value at Risk is an approach to quantify the amount of loss that a bank, a fund manager, an investor or a trading strategy might incur within the next n future days at a confidence level of e.g. 95% or 99%, provided that future returns are distributed like past returns. It is an important key figure in banking: Under the Basel II accord, banks are legally obligated to report their market risk in terms of Value at Risk on a daily basis, to assure that pre-set maximum losses won't be exceeded within given time horizons.

In addition, Expected Shortfall estimates the potential extent of damage for unlikely but possible extreme events (in terms of choice of α), where VaR is exceeded. Expected Shortfall is essential for describing the state space beyond VaR, e.g. when structuring finance products with insurance nature.

3.2.1. Application

We propose to apply Value at Risk and Expected Shortfall in discrete event simulation to describe the extent of state movements: VaR and CVaR can be employed to quantify the state changes observed in simulation experiments within set time spans Δt and at given confidence levels $1 - \alpha$. From this perspective, Value at Risk gives an impression of how far future states may depart from given start states in the predominant $1 - \alpha$ fraction of all cases. Expected Shortfall delivers the mean of states to expect when simulation dynamics develops further than normal, for the remaining α fraction of cases.

3.2.2. Generalisations and Extensions

In order to adapt the two metrics Value at Risk and Expected Shortfall to general purpose simulation, a number of generalisations and extensions were performed:

1. The various application areas of discrete event simulation will not necessarily attribute risk only to downward state movements as shown in figures 2a and 6. Following our reasoning in subsection 2.1, VaR and CVaR were extended to also report on the upper end resp. both ends of state distribution. Thereby not only downside risk (figure 2a), but also upside risk and outside risk (figures 2b and 2c) are now covered.

2. VaR and CVaR originally are forward-looking metrics, estimating risk with reference to the last observed system state. Generally, this state need not be of special interest in discrete event simulation. According to our proposals in subsection 2.2, VaR and CVaR were generalised to refer to the median, the minimum, the maximum and the mode (most frequent) states. In finance, VaR and CVaR unconditionally incorporate all observed state transitions for a future state estimation, regardless of the start state. For higher accuracy, we use only those observations starting from one of the four mentioned reference states when compiling the corresponding four state distributions. Therefore each of the state distributions reports exactly about one specific reference state context, and all other (distorting) state dynamics are filtered out.

3. VaR and CVaR were enhanced by state regions around reference states, as suggested in subsection 2.2. Thus, sufficient numbers of state movements can be included into the two distributions that describe dynamics starting from minimum and maximum states.

4. When observing a start state, there may be no state change event exactly after the set time span Δt. In these cases the target state can be selected as the preceding, the following or the nearest state after Δt, as expounded in subsection 2.2.

5. A further approach is abstraction from absolute target states in favour of relative state differences observed, leading to a re-naming of the Value at Risk metric to "Delta at Risk" (DaR) and of the Expected Shortfall metric to "Conditional Delta at Risk" (CDaR).

3.2.3. Presentation in Simulation Report

The DaR and CDaR metrics are based on identical recordings of state variable changes. Therefore, they are computed conjointly and are presented together in the simulation report, as shown in figure 7.

Delta at Risk and Conditional Delta at Risk

Name	Obs	Search Mode	Time Span	First Table	Second Table
Queue Length	7442	Previous	1.0 Hours	Delta at Risk	Conditional Delta at Risk

Start State	Value	Region	Effective	Lowest 1.0%	Lowest 2.5%	Lowest 5.0%	Lowest 10.0%	Highest 10.0%	Highest 5.0%	Highest 2.5%	Highest 1.0%
Minimum	25.0	+5.0	25.0 ... 30.0	36.0	37.0	38.15	40.0	54.0	55.0	55.0	56.54
Mode	47.0	+/-0.0	47.0 ... 47.0	-19.64	-16.1	-13.2	-9.4	30.0	33.2	38.0	39.0
Median	62.0	+/-0.0	62.0 ... 62.0	-23.0	-22.075	-21.15	-17.6	22.6	28.0	28.225	31.69
Maximum	100.0	-5.0	95.0 ... 100.0	-60.0	-59.0	-58.7	-51.0	-7.0	-7.0	-4.55	-3.42

Start State	Value	Region	Effective	Lowest 1.0%	Lowest 2.5%	Lowest 5.0%	Lowest 10.0%	Highest 10.0%	Highest 5.0%	Highest 2.5%	Highest 1.0%
Minimum	25.0	+5.0	25.0 ... 30.0	36.0	36.34959	36.88618	38.37398	54.97561	55.3252	55.65041	56.62602
Mode	47.0	+/-0.0	47.0 ... 47.0	-20.27119	-18.37288	-16.47458	-13.87288	34.19492	37.23729	38.50847	39.0
Median	62.0	+/-0.0	62.0 ... 62.0	-23.0	-22.51948	-22.03896	-20.63636	26.67532	28.77922	29.55844	31.0
Maximum	100.0	-5.0	95.0 ... 100.0	-60.0	-59.56338	-59.23944	-56.07042	-6.01408	-5.02817	-3.87324	-3.29577

top

Figure 7: Textual Presentation of Delta at Risk and Conditional Delta at Risk in Simulation Report

The "Delta at Risk and Conditional Delta at Risk" report section consists of two main tables, the upper one for Delta at Risk and the lower one for Conditional Delta at Risk. Each table has four lines of content, denoting the start states Minimum, Mode, Median and Maximum. The first four table columns list type, values, radii and regions of the reference states. Further four columns show the DaR resp. CDaR values for confidence levels 99%, 97.5%, 95% and 90%. These entries contain the lowest state deltas (shortfall deltas) observed after the set period Δt when starting at one of the four reference states and demanding one of the given confidence levels.

To also account for upside risk, the observed upper boundary states (α-quantiles) for the mirrored distribution and same confidence levels are stated. Outside risk is obtainable by combining downside risk and upside risk.

If the report format is set to HTML with graphics, DaR and CDaR are also displayed graphically, see figure 8. Each of the four charts refers to one start state. Per chart, four groups with two bars present the values for downside and upside Delta at Risk for each of the four confidence levels 99%…90%. The Conditional Delta at Risk level is displayed as a "gloriole" on top or below of each bar. The distance between the end of a bar (DaR) and the gloriole of the bar (CDaR) gives an impression of the difference between normal model dynamics and rare extreme dynamics.

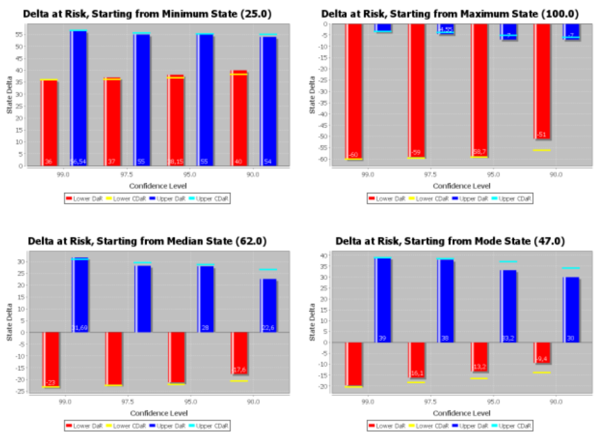

Figure 8: Graphical Visualisation of Delta at Risk and Conditional Delta at Risk in Simulation Report

3.2.4. Further Development

A lesson learned was that presentation of 64 outcomes (2 metrics * 4 reference states * 4 confidence levels * 2 downside/upside distribution ends) can be overwhelming and might even obstruct analysis. This is true for the textual presentation in the simulation report, but also applies to the four charts with altogether 32 bars and 32 glorioles. Another experience is that thinking in terms of relative state changes has advantages, but often it is easier to regard absolute states that are directly connected with representative states in the modelled system. A third insight is that state deviations into one direction are of interest, but it is at least as interesting to depict the *range* of states that can be expected as outcomes of a start state after a time span Δt and with a given confidence level. These three aspects led to development of a compact, combined chart of absolute state movement ranges, which is additionally included in the DESMO-J simulation report and displayed in figure 9.

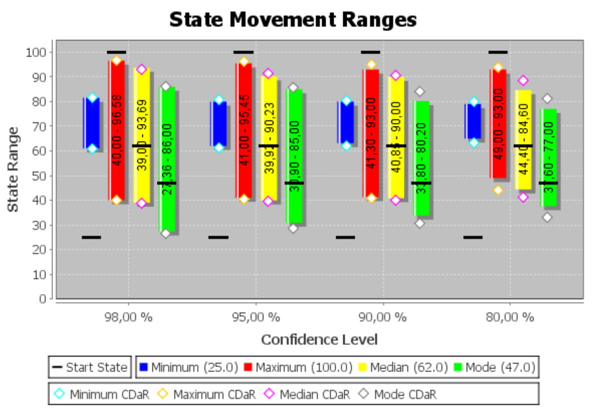

Figure 9: State Movement Ranges Chart in Simulation Report

The State Movement Ranges chart is divided into four areas, one area per confidence level. Every area consists of four bars, representing the absolute range of states that will be reached after Δt at the chosen confidence level. The bar ranges are symmetrical with respect to the boundary observations included into a bar: E.g. a bar in the 90% area starts at the 5th percentile and ends at the 95th percentile and therefore contains 90% of all observations. The start and end values are noted within each bar. Each of the four bars in an area refers to one of the reference states minimum, maximum, median or mode; the numerical values of reference states are displayed in the chart legend. These reference start states of dynamics are represented by black lines. A black line inside a bar (here at the median and the mode) indicates that future target states will be distributed around the start state. The two diamonds above and below a bar indicate the expected value of the remaining observations above resp. below the bar, which are not included in the bar themselves.

The state movement ranges chart resembles box-and-whisker plots, but should not be confused with them: a) A dynamics bar contains observations according to the desired confidence level (90%…99%), whereas a boxplot contains fixed 50% of observations. b) Black horizontal lines in state movement ranges charts stand for the start states of dynamics, whereas thicker horizontal lines within boxplots represent the median of observations. c) State movement ranges charts always contain exactly two diamonds, above and below a bar, representing the mean of those states that are not included in a bar. Boxplots may contain zero to multiple non-cumulated data points for outliers, and every circle symbol stands for exactly one outlier.

As an interpretation example, the following conclusions can be drawn from figure 9 (which traces back to the same queuing system that produced figures 3–5 and 7–8): Within Δt of one hour, dynamics will lead far away from the minimum queue length of 25, into a rather narrow range between 60 and 80 waiting clients. The target states from 60 to 80 correspond to approx. 25% of the state space (25 to 100 waiting clients). Dynamics starting from the maximum queue length of 100 are different: queue length may either remain at high levels or decrease down to around 40. Here downside dynamics within one hour covers more than 75% of the state space and thus has a higher variance resp. uncertainty than upside dynamics. In the most frequent case with a queue length of 47 clients, queue length rather grows than shrinks within one hour. Nevertheless, low states near the minimum are only reached from other lower states like the mode. Dynamics around the median state 62 seem to be distributed symmetrically. There are no surprising outliers in state dynamics, because the diamonds representing expected states in extreme cases are relatively near to the bars which represent usual dynamics.

3.3. Drawdown

In quantitative finance, the term *drawdown* denotes the extent of an interim loss of asset value, after a new peak of asset value was reached beforehand (Burghardt et al. 2003). In a broader sense, the term drawdown also relates to the whole time interval in which a setback of asset value is observed.

If the value of a portfolio fluctuates upwards in the long run, then every new peak of asset value will be followed by a drawdown, until the next higher peak is reached. Hence a portfolio will almost always be in a drawdown, since new peaks of portfolio value are rather rare events.

The main focus in quantitative finance is on the maximum drawdown that a trading strategy has ever undergone (Acar and James 1997, Chekhlov et al. 2000, Mendes and Leal 2003, Magdon-Ismail and Atiya 2004). This figure quantifies the maximum magnitude of interim losses a strategy has incurred in the past. It indicates how much initial capital or risk disposition an investor needs to pursue a strategy, provided that historical drawdowns are not exceeded by future drawdowns. If the initial capital or tolerance to interim losses is lower than the expected maximum drawdown of a strategy, then the strategy should not be followed, because all invested capital may be lost or the investor might withdraw too early from a long-term strategy due to nervousness.

A further (but less popular) key figure is the average drawdown, which is computed as the mean of all drawdowns observed. It can give an impression whether an incurred present drawdown is typical or exceptional and thus should be monitored carefully. Other key figures are the average length and maximum length of all drawdown phases. They are applied to compare the duration of drawdown phases with investors' patience to wait for asset value recovery. For similar reasons, sometimes the time span from the instance of maximum drawdown to new peaks is considered.

Compared to e.g. Value at Risk, drawdown is weakly covered in academic literature. This may be associated with the circumstance that there are not many starting points to handle drawdown analytically. Drawdown rather describes an aspect of dynamic behaviour and commonly is not used in conjunction with statistical distributions or moments (in contrast to e.g. returns). Nevertheless drawdown is established in professional practice like futures trading or funds of funds management.

3.3.1. Generalisation

Against the backdrop of sometimes inconsistent and vague utilisation in finance, we generalise and re-define the term drawdown for application in discrete event simulation (see figure 10): A *drawdown phase* is a time span in which the value of a certain state variable declines and recovers with respect to a preceding maximum state. The difference between the preceding maximum state and the lowest state observed within the drawdown phase is the *drawdown extent*. The time instant at which (the last occurrence of) this local minimum state was observed is the *drawdown instant*. The time span between the start of a drawdown phase and the drawdown instant is denoted *drawdown time*. The time span between the drawdown instant and the end of a drawdown phase is named *drawdown recovery time*. A drawdown phase ends with the first observation of a state equal or higher than the preceding maximum at which the drawdown phase started. A drawdown phase starts with the last observation of the preceding global state maximum. One particular *drawdown* is specified by start of the drawdown phase, drawdown time, drawdown recovery time and the drawdown extent. It can be represented by a quadruple $q \in \mathbb{T} \times \mathbb{I} \times \mathbb{I} \times \mathbb{S}$, where $\mathbb{T}$ is the data type of (simulation) time instants, $\mathbb{I}$ is the data type for simulation time intervals and $\mathbb{S}$ is the data type of the variables' states (differences). The entirety of all definitions above is called *drawdown concept*.

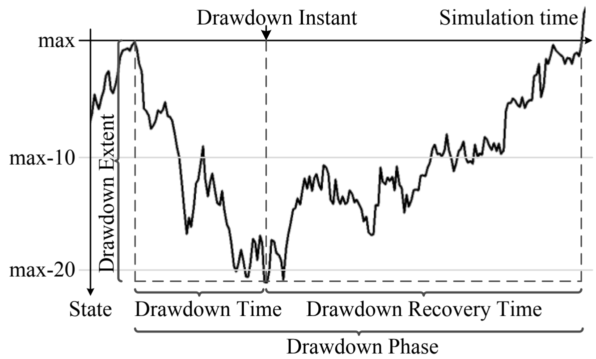

Figure 10: Illustration of the Drawdown Concept

3.3.2. Application

We propose to apply the drawdown concept in discrete event simulation to characterise observed dynamics of state variables. Drawdown time and drawdown extent quantify susceptibility in terms of (counter) movement towards lower states, whereas drawdown recovery time describes regenerative behaviour in the analysed system. By extending the drawdown concept as introduced in the following subsection, additional insight into the dynamics of a simulation variable can be gained.

3.3.3. Extensions and Further Development

The drawdown concept embodies a risk type as shown in subsection 2.1, figure 2f: (un)desired states in a growth process are path dependent. Former global maximum states that were welcome in the past may precede newer higher maximums in the present. Hence, the same state observation that was desired in the past may pose a risk when a growth process continues.

With respect to reference states, the drawdown concept defined above refers to the previous maximum as reference state (cf. subsection 2.2). It is obvious to transfer the drawdown concept to reverse contraction processes, by referring to the previous minimum. In this case, counter movements towards higher states can be regarded risky. In order not to confuse terminology, we apply a reverse naming scheme and incorporate *runup phases*, *runup extent*, *runup instant*, *runup time*, *runup recovery time* and particular *runups* into the *runup concept*.

Logically, the drawdown concept is restricted to growth processes and the runup concept to contraction processes: for example, an ever-growing process with states that do not fall below the initial (minimum) state has only one runup, which is the process itself.

The next step is to extend the drawdown/runup concept to processes in steady states as shown in figures 2a and 2b. We accomplish this by utilising the median and mode state as reference states, instead of previous maximum or previous minimum, as before. If the expression "max" in figure 10 is replaced by e.g. "median", then the drawdown concept will characterise all dynamics below the median and the runup concept will describe all dynamics above the median. With median or mode as reference states, the drawdown and runup concepts can be combined to treat dynamics below and above the reference state at the same time.

The next conceptual step is to abandon the separation into drawdown concept and runup concept for steady state processes fluctuating around the median or mode (or any other) state. In the context of reference states that are not previous minimum or previous maximum, it makes sense to unify both concepts to an *excursion concept*, with *excursion phases*, *excursion extent*, *excursion instant*, *excursion time*, *excursion recovery time* and particular *excursions*. Former drawdowns and runups are still recognisable by the algebraic sign of the fourth component of an excursion quadruple: excursions with positive extent are runups and excursions with negative extent are drawdowns. The advantage of this unification is standardisation of methodology, algorithms and terminology, without inconvenient case differentiations.

As a last step, the remaining drawdown and runup concepts that still refer to previous maximum and previous minimum may be generalised to excursions, too, harmonising terminology and technical implementation overall.

3.3.4. Presentation in Simulation Report

When the excursion metric is declared for a state variable, the dynamics of this variable is recorded during simulation and processed at the end of an experiment, because median and mode state are not known beforehand. Here, excursion quadruples $q \in \mathbb{T} \times \mathbb{I} \times \mathbb{I} \times \mathbb{S}$ are extracted from the recording. They contain start of the excursion phase (data type $\mathbb{T}$), excursion time span and excursion recovery time span (both of data type $\mathbb{I}$) and excursion extent (data type $\mathbb{S}$). For faster and more convenient processing in DESMO-J, it is required that data types $\mathbb{I}$ and $\mathbb{S}$ have a representation as subtypes of Java `Number`.

Since the *relative* state pathway of excursions is of more interest than their exact start time instant, the last three components of excursions are central and covered in the following analysis. For the time being, the first component indicating the exact start time is abstracted from.

The set of collected excursion quadruples of a state variable is analysed in different ways, to account for various aspects. For a first overview, the top n (most relevant) excursions from and to median, mode, minimum, maximum, previous minimum and previous maximum states are listed in descending order, with respect to excursion extent, excursion time, excursion recovery time and excursion total time (figure 11).

Excursions

Name	Obs	Minimum	Mode	Median	Maximum
Queue Length	7442	25.0	47.0	62.0	100.0

…

Excursions from and to Mode

Observed Excursions	Excursions with negative Extent	Excursions with positive Extent
235.0	116.0	119.0

Top 3 Excursions from and to Mode, ordered by Extent

>Excursion Extent	Excursion Time	Excursion Recovery Time	Excursion Total Time
53.0	1.08647	1.38311	2.46958
52.0	1.32132	1.03468	2.356
38.0	0.50326	0.39434	0.8976

Top 3 Excursions from and to Mode, ordered by Excursion Time

Excursion Extent	>Excursion Time	Excursion Recovery Time	Excursion Total Time
52.0	1.32132	1.03468	2.356
53.0	1.08647	1.38311	2.46958
38.0	0.50326	0.39434	0.8976

…

Figure 11: Segment of Textual Listing of the most relevant Excursions in Simulation Report

If report format is set to HTML with graphics, the distributions for the three independent components excursion extent (figure 12), excursion time and excursion recovery time are approximated by kernel density estimations with selectable kernels and bandwidths.

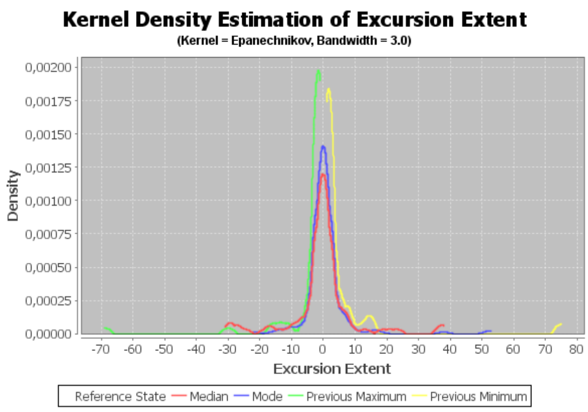

Figure 12: Graphical Visualisation of Excursion Distributions as Kernel Density Estimations in Simulation Report

The kernel density estimations shown above give an independent view of each excursion component's distribution. To convey an integrated impression of model dynamics, it is helpful to provide a view on all excursion tuples related to one reference state at once. For this purpose, we provide a scatter plot (per reference state) that maps excursion time to the x-axis, excursion recovery time to the y-axis and absolute excursion extent to shapes and colours (figure 13). Blue upward triangle symbols represent upward excursions (runups) and red downward triangles stand for downward excursions (drawdowns).

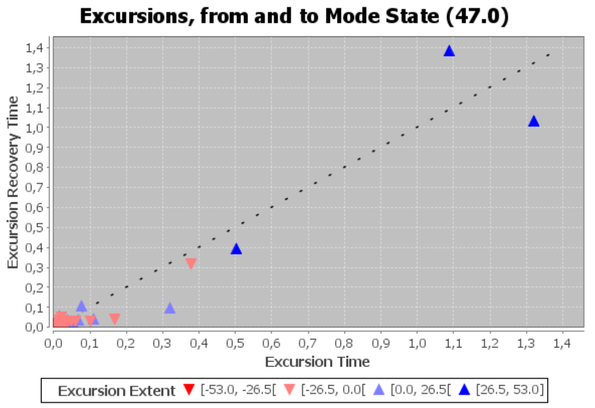

Figure 13: Graphical Visualisation of Excursions in Simulation Report

The scatterplot overview shown above is supportive for analysis of location, distribution and relationships of excursions from a bird's eye view. As an aggregated view, it cannot contain information about the specific pathways of single excursions: this data is not contained in excursion tuples. But since the basic recordings of state variable dynamics are accessible at report generation time, it is easy to cut the whole time series of state variable dynamics into pieces (one per excursion) and to present these parts in further charts (one chart per reference state, figure 14).

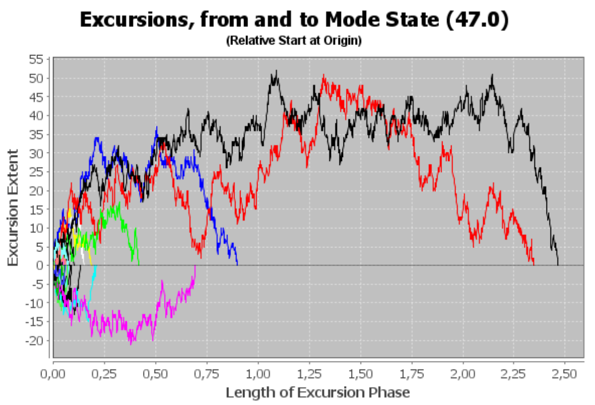

Figure 14: Graphical Visualisation of all Excursion Pathways in Simulation Report

Here, all excursions are plotted together into one coordinate system. The start of each excursion is set to the origin of ordinates. The time span since start of an excursion is shown on the x-axis and the relative state distance from the start state is indicated on the y-axis. Naturally, runups are plotted above the x-axis and drawdowns are plotted below it. In stochastic experiments, most excursion phases will be rather short with low excursion extent, due to noise that causes small arbitrary movements around previous states. But it is interesting to analyse longer excursions phases: How do runups and drawdowns relate with respect to a) length, b) extent and c) quantity? Is there a common shape of excursions? How are excursions distributed: Do only few “outlier” excursions account for large parts of dynamics, or is there a dense distribution of mid-length excursions? How stable are excursions: Do they consistently remain far away from the x-axis or do they temporarily collapse almost back to the x-axis, before departing to more distant states again; etc.

Figure 14 facilitates analysis of excursions as a whole. When it comes to examination of excursion recovery, a slightly different presentation is rewarding: All excursions plotted in figure 14 are shifted parallel to the x-axis to the point where the excursion instant is on the y-axis. In this way all excursions are centred (not necessarily symmetrically) around the y-axis, with their highest absolute extent on the y-axis (figure 15).

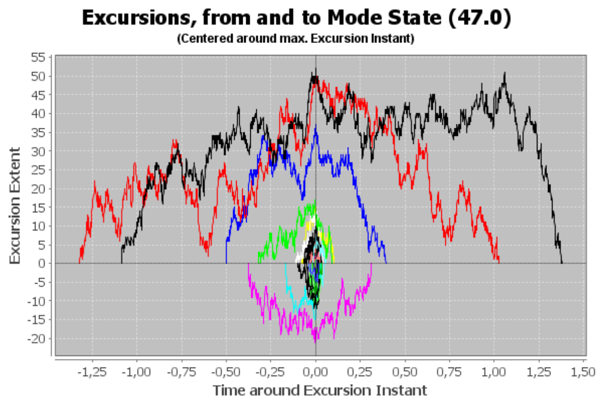

Figure 15: Graphical Visualisation of Excursion Recovery Pathways in Simulation Report

As a result, recovery dynamics back to reference states can be explored when investigating the right side of the y-axis and dynamics from reference states towards maximum excursion instants can be explored on the left side of the y-axis. Objects of study may be internal symmetry of excursions or typical shapes on the left and right hand side of the y-axis

As an interpretation example, it can be concluded from figures 14 and 15 that the underlying queuing system (cf. figure 5) has a large number of excursion phases shorter than a quarter of an hour. Starting at the mode reference state, these short-term excursions do not change the queue length to an extent further than ±15 clients. Only five excursions (clearly visible in figure 13) account for the remaining mid-term dynamics of the system. These mid-term excursion phases have a comparable length before and after the excursion instant. It is noticeable that there is only one significant drawdown of lower extent, but four significant runups, three of them with high extent. This supports the conclusion from figure 9 (state movement ranges), that dynamics around the mode state is distributed asymmetrically and has a preference for higher states.

4. SUMMARY AND OUTLOOK

In this paper, we propose a conceptual procedure to integrate methods of specific application domains into general purpose discrete event simulation. In particular, four established risk metrics from quantitative finance have been generalised: semivariance, Value at Risk, Expected Shortfall and drawdown. Quantitative finance risk notions like uncertainty, downside risk and drawdown risk have been extended to the risk types upside risk, outside risk, transition risk, critical state risk, runup risk and excursion risk. The standard use of mean and previous maximum as reference states has been extended to median, mode, minimum, maximum and previous minimum reference states. State regions and flexible target states provide further application options. These generalised risk metrics have been implemented in our open source discrete event simulation framework DESMO-J, utilising the JFreeChart charting library. A number of chart types have been employed to visualise model dynamics in the simulation report, particularly bar charts, extended time series charts, interval bar charts, histograms, scatter plots and line charts.

Table 1 gives an overview on the current status of implementation of the four pre-mentioned risk metrics. It shows the extent of generalisation introduced in terms of risk type and reference state as well as the chart types provided for visualisation.

Generalisations and visualisations currently implemented are denoted with a checkmark. The term "n/a" marks combinations of risk metrics with generalisation or visualisation types that do not seem feasible in principle. The letters a) to f) before risk types and reference states refer to the corresponding passages of subsection 2.1 resp. subsection 2.2.

As can be seen from table 1, considerable functionality has already been implemented; however some blank spots remain at the current stage. Further development is planned on the following items (denoted in the entries of table 1):

1. Extend semivariance to other constant reference states than mean.
2. Extend semivariance to dynamically reference previous extreme states, in order to assess drawdown and runup risk.
3. Extend Value at Risk and Expected Shortfall to reference mean as well as previous extreme states in order to assess drawdown and runup risk.
4. Visualise Value at Risk and Expected Shortfall in a chart similar to figure 6.
5. Overlay the basic time series of observed states with a rotated chart similar to the aforementioned item 4.
6. Extend drawdown (resp. excursions) to reference states mean, minimum and maximum.
7. Extend Value at Risk, Expected Shortfall and drawdown conceptually to transition risk.
8. Extend drawdown conceptually to outside risk and critical state risk.
9. Extend semivariance to a generalised deviation concept, providing additional assessment of outside, transition and target risk.

Table 1: Implementation Status of Risk Metrics and Starting Points for Further Development

	Generalisation	Semivariance	Value at Risk	Expected Shortfall	Drawdown
Risk Type (Sect. 2.1)	a) Downside Risk	✓	✓	✓	✓
	b) Upside Risk	✓	✓	✓	✓
	c) Outside Risk	9.	✓	✓	8.
	d) Transition Risk	9.	7.	7.	7.
	e) Critical State Risk	9.	n/a	n/a	8.
	f) Drawdown Risk	2.	3.	3.	✓
	Runup Risk	2.	3.	3.	✓
	Excursion Risk	n/a	n/a	n/a	✓
Reference State (Sect. 2.2)	Mean	✓	3.	3.	6.
	a) Median	1.	✓	✓	✓
	b) Mode	1.	✓	✓	✓
	c) Minimum	1.	✓	✓	6.
	c) Maximum	1.	✓	✓	6.
	d) Previous Maximum	2.	3.	3.	✓
	d) Previous Minimum	2.	3.	3.	✓
	State Regions	n/a	✓	✓	n/a
	Flexible Target State	n/a	✓	✓	n/a
Visualisation	Bar Chart	✓	✓	✓	n/a
	Extended Time Series Chart	✓	5.	5.	
	Interval Bar Chart	n/a	✓	✓	
	Histogram	n/a	4.	4.	✓
	Scatter Plot	n/a			✓
	Line Chart	n/a	4.	4.	✓

The added value of this contribution is to apply and visualise generalised financial risk metrics in discrete event simulation, embedded into a conceptual frame. By this approach, additional means for assessment of model dynamics can be obtained, complementing standard descriptive simulation statistics: Extended semivariance supports evaluation of steady state phases; generalised Value at Risk and Expected Shortfall provide quantification of (un)desired state movement extents; generalised drawdown facilitates characterisation of state variable dynamics. In this way, advanced analysis methods are made available for further use in simulation application fields that could not use financial risk metrics before, due to dissimilar risk notions. We conclude that generalising and transferring methods from specific simulation application domains into general purpose discrete event simulators, as we have done here, may benefit and hopefully inspire a wide range of further application fields.

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